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ABSTRACT

This is the teacher's manual to accompany part 2 of the workbook for grade 7 pupils in the Comprehensive Instructional Management System, a teacher-designed unified mathematics curriculum for kindergarten through grade 7. It is composed of a curriculum, a testing component, and a computer-managed reporting system. The curriculum is organized into modules of similar skills or understandings from 14 topics. Each module has: a suggested period of duration for student mastery; a listing of new skills to be mastered by the student in the form of performance objectives; a section devoted to vocabulary, terms, and symbols; getting-started activities on prerequisite skills in preparation for undertaking new objectives; strategies for teaching the new skills; and student activities for providing reinforcement for the newly learned objective. After the sequence of modules and content by topic are displayed, teaching suggestions for 58 modules on over 130 performance objectives are given. The content is from the topics of: numbers and numeration; addition; subtraction; fractions; geometry; measurement; algebraic concepts; graphs, probability, and statistics; and problem solving, plus review. Pages from the student's workbook are interspersed with the suggested activities. (MNS)

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CIMS

COMPREHENSIVE INSTRUCTIONAL MANAGEMENT SYSTEM

A Cyclical Mathematics Curriculum

PART 2—TEACHER

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FOREWORD

The Comprehensive Instructional Management System (CIMS) is a unified mathematics curriculum for grades K-7. The curriculum is cyclical in design, and reflects the most recent New York City and New York State guidelines.

Utilizing the experience of our system's teachers, we have developed a teacher-designed curriculum that includes a rich selection of classroom-tested successful mathematics activities. The major goal of the CIMS mathematics project is to provide the classroom teacher with a well-structured, objective-based curriculum. All objectives are supported by a variety of strategies. Student mastery of these objectives is readily diagnosed through the system's testing component.

The curriculum format is structured to allow the teacher to determine the ability level of the students in the topic under study, and to permit the students to move forward at their individual rates. Each module of instruction provides the teacher with a set of related performance objectives. These objectives are supported by activities which probe and develop the students' readiness to undertake new skills and concepts. Further reinforcement is given to mastery of the objectives through accompanying student workbooks.

It is hoped that this curriculum will not only assist teachers in developing vital mathematics instruction, but will serve to expand and enrich students' mathematics skills.

Charlotte Frank

Charlotte Frank,
Executive Director
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ACKNOWLEDGMENTS

The Comprehensive Instructional Management System (CIMS) was developed as a project of the Division of Curriculum and Instruction, Charlotte Frank, Executive Director. Dr. Peter Negroni, Community School Superintendent CSD 12; Dr. Harvey Garner, Community School Superintendent CSD 18; Carlos Edwards, Community School Superintendent CSD 23, and Joseph Pacheco, Community School Superintendent CSD 32 supervised the overall development of the CIMS program within their respective districts.

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INTRODUCTION

The Comprehensive Instructional Management System (CIMS) is a teacher-designed educational delivery system composed of a curriculum, a testing component and a computer managed reporting system.

Designed to support mathematics instruction in the classroom, CIMS provides a comprehensive cyclical mathematics curriculum for grades K through 7. Each manual describes the curriculum for a single level or grade. Teacher curriculum developers from elementary, intermediate and senior high schools utilized the most current New York City and New York State guidelines in formulating the curriculum's modules and sequence.

Curriculum Topics

The CIMS mathematics curriculum has been organized into the following 14 topics:

.01	Numbers and Numeration
.02	Addition of Whole Numbers
.03	Subtraction of Whole Numbers
.04	Multiplication of Whole Numbers
.05	Division of Whole numbers
.06	Fractions
.07	Decimals
.08	Integers
.09	Geometry
.10	Measurement
.11	Algebraic Concepts
.12	Graphs, Probability, Statistics
.13	Problem Solving
.14	Review

Modules of Instruction

The CIMS curriculum is organized into modules or groupings of similar skills or understandings. Each module has a suggested period of duration for student mastery, a listing of new skills to be mastered by the student in the form of performance objectives, a section devoted to vocabulary, terms and symbols, getting started activities which serve as prerequisite skills in preparation for undertaking new objectives, strategies for teaching the new skills and student activities for providing reinforcement for the newly learned objective. Note that the lesson duration is included only as a guideline to the teacher for planning purposes. The actual time required for each module will vary from class to class and from child to child depending on the needs of the students and the teacher's judgment.

Priorities of Performance Objectives

There is an order of priority to be recognized in teaching the content presented in the CIMS NEW YORK CITY MATHEMATICS CURRICULUM. Performance objectives with the greatest priority are so basic that unless the student acquires a satisfactory degree of competency in them, the student cannot progress. Such topics are marked A and mastery by all pupils is essential.

Performance objectives marked B are required to be presented to all students. Mastery is not expected at this time. These objectives will be developed further and reinforced later.

Performance objectives marked C are of special interest. These should be taught to increase positive attitudes towards mathematics.

Summary: A - Mastery of this objective is essential for progress. Opportunities for reinforcement should be provided throughout the school year.

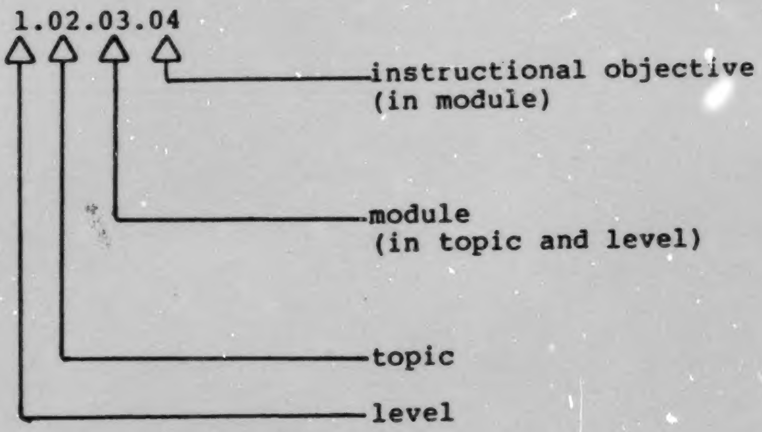
B - These objectives are to be taught to all pupils. Mastery is not essential at this time because the topics will be further developed and reinforced.

C - Objectives of special interest.

Topic Tab Key

01. Numbers and Numeration
02. Addition of Whole Numbers
03. Subtraction of Whole Numbers
04. Multiplication of Whole Numbers
05. Division of Whole Numbers
06. Fractions
07. Decimals
08. Integers
09. Geometry
10. Measurement
11. Algebraic Concepts
12. Graphs, Probability, Statistics
13. Problem Solving
14. Review

Module Coding Key


1.02.03.04
instructional objective
(in module)
module
(in topic and level)
topic
level

INSTRUCTIONAL SEQUENCE DESCRIPTION

An instructional sequence is given in list form on the following pages for this level. The sequence has been arranged to reflect what is considered the most effective presentation of the mathematics curriculum. Each module in the list is described by its unit number, module number, title, and page location.

The CIMS tests are integrated within the manual sequence. The test number is located in the first column. Each test is designed to assess priority objectives from the preceding modules. The tests are used to determine student mastery of specific performance objectives within these modules and appear in a multiple choice format. Each performance objective code number preceded by an * indicates that the performance objective is not tested on a CIMS test.

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INSTRUCTIONAL SEQUENCE - LEVEL 7

TEST NUMBER	UNIT NUMBER	MODULE NUMBER	TITLE	PAGE NUMBER
	1	7.11.01	Translating arithmetic word phrases into numerical phrases	1
	2	7.11.02	True and false numerical statements	2
	3	7.14.01	Numbers through billions	4
7.01	4	7.02.01	Addition of whole numbers	5
	5	7.01.01	Palindromes	6
	6	7.03.01	Subtraction of large numbers	7
	7	7.13.01	Solving a simpler but equivalent problem	8
7.02	8	7.04.01	Multiplication of large numbers	9
	9	7.05.01	Express a remainder as a fraction	11
	10	7.01.02	Divisibility rules: 2, 5, 10	12
7.03	11	7.01.03	Divisibility rules: 3, 4, 6, 9	13
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	13	7.09.02	Vertical angles	16
7.04	14	7.09.03	Triangles	17
	15	7.07.01	Decimal relationships	19
	16	7.07.02	Addition of decimals	21
	17	7.07.03	Subtraction of decimals	22
	18	7.13.02	Working backwards	23
	19	7.14.02	Fractional relationships - mixed numbers	24
	20	7.06.01	Advanced fraction computations and concepts	25
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	22	7.06.03	Ratio and proportion	27
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INSTRUCTIONAL SEQUENCE - LEVEL 7

TEST NUMBER	UNIT NUMBER	MODULE NUMBER	TITLE	PAGE NUMBER
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	26	7.10.03	Metric units	35
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INSTRUCTIONAL SEQUENCE - LEVEL 7

TEST NUMBER	UNIT NUMBER	MODULE NUMBER	TITLE	PAGE NUMBER
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7.09	51	7.08.02	Multiplication of signed whole numbers.....	68
	52	7.08.03	Division of signed whole numbers...........	69
	53	7.09.06	Parallel lines and their properties........	70
	54	7.09.07	Angles of a polygon........................	72
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7.03.01 Subtraction of large numbers............ 7

Topic 04: Multiplication of Whole Numbers

7.04.01 Multiplication of large numbers......... 9

Topic 05: Division of Whole Numbers

7.05.01 Express a remainder as a fraction....... 11

Topic 06: Fractions

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7.06.03 Ratio and proportion.................... 27
7.06.04 Meaning of percent...................... 46
7.06.05 Verbal problems involving percent....... 47

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7.13.03 Guess and check......................... 50
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7.14.05 Positive and negative integers.......... 66

Unit 25

7.10.02 COMPUTATION WITH CUSTOMARY MEASUREMENT
(3 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

*7.10.02.01A compute the sum of different units of length, time, weight, liquid measure; e.g.,

10 gal. 2 qts.
+6 gal. 1 qt.
16 gal. 3 qt.

*7.10.02.02A compute the difference of a measurement given in linear units, time units, weight units, or liquid volume units; e.g.,

5 weeks 4 days
-3 weeks 2 days
2 weeks 2 days

7.10.02.03A

compute the product of a measurement given in linear units, time units, weight units or liquid volume units;

e.g.,

$$\begin{array}{r} 6 \text{ lbs. } 10 \text{ oz.} \\ \underline{\times 2} \\ 12 \text{ lbs. } 20 \text{ oz} = 13 \text{ lbs. } 4 \text{ oz.} \end{array}$$

7.10.02.04A

compute the quotient of a measurement given in linear units, time units, weight units, or liquid volume units;

e.g.,

10 ft. 6 in. ÷ 2 = 5 ft. 3 in.

7.10.02.05A

solve verbal problems related to skills learned in this module.

e.g.,

A carpenter attaches a piece of wood 6 ft. 4 in. to another piece 2 ft. 6 in. How long are the boards together?

$$\begin{array}{rr} 6 \text{ ft.} & 4 \text{ in.} \\ \underline{+\ 2 \text{ ft.}} & \underline{6 \text{ in.}} \\ 8 \text{ ft.} & 10 \text{ in.} \end{array}$$

7.10.02

VOCABULARY

TERMS

customary measurement
liquid volume
linear

GETTING STARTED

Prepare two sets of index cards with a measurement on one set and its equivalent on the other.

e.g.,

1 ft.	12 in.
1 qt.	32 oz.

Number the back of each card.

Place both sets of cards face down on a desk. Play "Concentration" by asking the students to take turns turning over two cards. If the cards match, the cards remain face up, the student gets a point and goes again.

Continue to play until all cards are turned over.

Prepare an oak tag or display board with various measurements on it.

e.g.,

1 yd. = ___ ft.
1 ft. = ___ in.
1 gal. = ___ qt.
1 qt. = ___ pt.
1 pt. = ___ cup
1 cup = ___ oz.
1 year = ___ weeks
1 week = ___ day
1 day = ___ hrs.
1 hr. = ___ min.
1 lb. = ___ oz.

Ask various students to help fill in the equivalent measurements. Ask the students to copy the oak tag/display board into their notebooks.

7.10.02

7.10.02.01 **compute the sum of different units of length, time, weight and liquid measure**

STRATEGY 1: Say, "It takes Sue 1 hour and 35 minutes to do her homework. It takes her sister Nancy 1 hour and 55 minutes to do her homework. How can we figure out how long it took them altogether to do their homework?"

Ask a student to come to the chalkboard and set up the example.

e.g.,

$$\begin{array}{r} 1\text{ hr. } 35\text{ min.} \\ +\ \underline{1\text{ hr. } 55\text{ min.}} \end{array}$$

Ask another student to solve the problem. (2 hr. 90 min.) Say, "We have 90 minutes. What other way can we say 90 minutes?" (1 hour and 30 minutes.)

Write on the chalkboard, e.g., 2 hr. 90 min. = 3 hr. 30 min.

Repeat this procedure using various examples.

STRATEGY 2: Ask one student to measure the length of the room and another to measure the width. Ask the students to put their measurements on the chalkboard, e.g., 20 ft. 7 in, 14 ft. 8 in. Say, "If I wanted to put edging around those two sides, how much edging should I buy?"

Ask a student to write the example to solve the problem.

e.g.,

$$\begin{array}{r} 20\text{ ft. } 7\text{ in.} \\ +\ \underline{14\text{ ft. } 8\text{ in.}} \\ 34\text{ ft. } 15\text{ in.} \end{array}$$

Say, "We have 15 inches. What other way can we say 15 inches? (1 foot 3 inches.) We add the 1 foot to the 34 ft. and have 3 inches leftover or 35 feet 3 inches."

Repeat this procedure asking the students to measure various items around the room.

7.10.02

7.10.02.02 **compute the difference of a measurement given in linear units, time units, weight units or liquid volume units and a whole number**

STRATEGY 1: Select two students in the class and measure them. Write the measurements on the chalkboard.

e.g.,
Adam - 4 ft. 10 in.
David - 5 ft. 2 in.

Say, "How much taller is David than Adam?" Ask another student to write this as an example on the chalkboard to solve.

e.g.,
  5 ft. 2 in.
- 4 ft. 10 in.

Say, "We cannot subtract 10 inches from 2 inches. We must exchange 1 foot for 12 inches." Rewrite the example as
  4 ft. 14 in.
- 4 ft. 10 in.

Ask a student to find the difference. (4 inches)

Repeat this procedure using similar problems.

STRATEGY 2: Prepare a scale. Weigh two volunteers. Write their weights on the chalkboard.

e.g.,
Danny - 86 lb. 3 oz.
Frank - 72 lb. 9 oz.

Say, "How much more does Danny weigh than Frank?" Ask a student to come to the chalkboard and compute the difference. Since 16 ounces = 1 pound, we must regroup to subtract.

e.g.,
  85   19
  ~~86~~ lb. ~~3~~ oz.
- 72 lb. 9 oz.
  13 lb. 10 oz.

Repeat this procedure using various examples.

7.10.02

7.10.02.03 **compute the product of a measurement given in linear units, time units, weight units or liquid volume units**

STRATEGY 1: Say, "Sara is babysitting every day for one week for her neighbor. She is babysitting for 1 hour and 30 minutes each day for 7 days. How many hours, in total, is Sara babysitting?"

Ask a student to come to the chalkboard and write this as a multiplication problem.

e.g.,

$$\begin{array}{ll} 1\text{ hr.} & 30\text{ min.} \\ \underline{\times\phantom{\text{ hr.}}} & \underline{7\phantom{\text{ min.}}} \\ 7\text{ hr.} & 210\text{ min.} \end{array}$$

Say, "How can we rewrite 210 minutes? (210 minutes = 3 hr. 30 min.) How can we rewrite the product?" (7 hr. 3 hr. 30 min. = 10 hr. 30 min.)

Repeat this procedure using various examples.

STRATEGY 2: Cut five strips of oak tag measuring 3 feet 6 inches each. Say, "I have five strips. How much stripping do I have altogether?" Ask a student to write this as a multiplication example on the chalkboard.

e.g.,

$$\begin{array}{ll} 3\text{ ft.} & 6\text{ in.} \\ \underline{\times\phantom{\text{ ft.}}} & \underline{5\phantom{\text{ in.}}} \\ 15\text{ ft.} & 30\text{ in.} \end{array}$$

Say, "How can we rewrite 30 inches? (2 ft. 6 in.) How can we rewrite the product?" (15 ft. + 2 ft. 6 in. = 17 ft. 6 in.)

Repeat this procedure using various lengths.

250

24

7.10.02.04 **compute the quotient of a measurement given in linear units, time units, weight units or liquid volume units ~~and a whole number~~**

STRATEGY 1: Say, "The perimeter of a room is 21 feet 4 inches. I want to find the length of one side of the room. How can I do this?" Ask a student to come to the chalkboard and write out a division example.

e.g.,

$$21 \text{ ft. } 4 \text{ in.} \div 4 = \quad \text{or} \quad 4\,\overline{)\,21 \text{ ft. } 4 \text{ in.}}$$

Say, "How many times does four go into twenty-one feet? (four times with 1 left over) We must exchange the 1 foot and make it into inches to divide. How many inches do we know have? (4 in. + 12 in. = 16 in.) How many times does 4 go into 16 inches?" (4 times)

e.g.,

$$\begin{array}{r} 5 \text{ ft. } \;\; 4 \text{ in.} \\ 4\,\overline{)\,20 \text{ ft. } 16 \text{ in.}} \end{array}$$

Repeat this procedure using various examples.

7.10.02.05 **solve verbal problems related to skills learned in this module**

STRATEGY 1: Write a verbal problem involving measurement for each student on an index card.

e.g.,

> Jane needs 2 yd. 1 ft. of material to make a dress. How much material will she need for 4 dresses?

Give an index card to each student. Ask the students to write out the example for each word problem and compute.

Repeat this procedure by having the students switch cards.

7.10.02

STRATEGY 2: Say, "Bob and Richie each have paper routes. It takes Bob 1 hour 40 minutes to complete his route. It takes Richie 2 hrs. 10 min. to complete his route. How much longer does it take Richie than Bob to complete his paper route?"

Ask the students to determine the correct operation and write an example.

e.g.,

$$\begin{array}{rl} & 2\text{ hrs. } 10\text{ min.} \\ - & \underline{1\text{ hr. }\ \ 40\text{ min.}} \end{array}$$

Say, "We cannot subtract 40 minutes from 10 minutes. We must take 1 hour and exchange it for 60 minutes."

Rewrite the problem.

e.g.,

$$\begin{array}{rl} & \overset{1}{\cancel{2}}\text{ hrs. } \overset{70}{\cancel{10}}\text{ min.} \\ - & \underline{1\text{ hr. }\ \ 40\text{ min.}} \end{array}$$

Ask the students to solve the problem.

Repeat this procedure using various examples.

P.O. 7.10.02.01

NAME: ______________________ CLASS: __________ DATE: __________

Compute each sum.

Example:

```
   3 ft.  6 in.
+  4 ft.  8 in.
   ------------
   7 ft. 14 in.
       1 ft. 2 in. = 8 ft. 2 in
```

Note: in the example, 14 is struck through and replaced by "1 ft. 2 in." beneath it.

①

```
   4 hrs.  25 min.
+  2 hrs.  40 min.
   ---------------
```

②

```
   4 ft.  7 in.
+  2 ft.  6 in.
   ------------
```

③

```
   3 wks.  4 days
+  1 wk.   5 days
   --------------
```

④

```
   3 lb. 12 oz.
+  2 lb.  7 oz.
   ------------
```

⑤

```
   6 yd.  2 ft.
+  2 yd.  2 ft.
   ------------
```

⑥

```
   3 hrs. 46 min.
+  2 hrs. 55 min.
   --------------
```

⑦

```
   14 lb.  9 oz.
+   5 lb. 11 oz.
   -------------
```

P.O. 7.10.02.02

NAME: ______________________ CLASS: ________ DATE: ________

Compute each difference.

Example:

$\overset{4}{\cancel{5}}$ ft. $\overset{15}{\cancel{3}}$ in.
- 2 ft. 4 in.
2 ft. 11 in.

1.
6 wks. 4 days
- 3 wks. 6 days

2.
9 hrs. 25 min.
- 3 hrs. 50 min.

3.
17 lbs. 4 oz.
- 12 lbs. 15 oz.

4.
2 yds. 1 ft.
- 1 yd. 2 ft.

5.
6 qts. 1 pt.
- 4 qts. 2 pts.

6.
8 ft. 6 in.
- 4 ft. 10 in.

7.
10 gal. 2 qts.
- 5 gal. 3 qts.

P.O. 7.10.02.03

NAME: ______________________ CLASS: ________ DATE: ________

Compute the products.

Example:

6 ft. 4 in.
x5

30 ft. 20 in. =
1 ft. 8 in. =
31 ft. 8 in.

(1.) 6 qts. 1 pt.
x6

(2.) 2 wks. 5 days
x3

(3.) 2 ft. 9 in.
x3

(4.) 4 qts. 2 pts.
x5

(5.) 3 yds. 2 ft.
x6

(6.) 8 lbs. 10 oz.
x3

(7.) 5 gal. 3 qts.
x4

P.O. 7.10.02.04

NAME: ______________________ CLASS: ________ DATE: ______

Compute the quotients.

3) ~~7~~ ft. ~~6~~ in. = 6 ft. 18 in. 2 ft. 6 in. 3) 6 ft. 18 in. = 2 ft. 6 in.	1. 5) 11 yd. 2 ft.
2. 2) 5 wk. 3 days	3. 2) 7 yd. 1 ft.
4. 5) 11 ft. 8 in.	5. 3) 7 gal. 2 qts.
6. 2) 5 gal. 2 qt.	7. 6) 13 hrs. 12 min.

P.O. 7.10.02.05

NAME: ______________________ CLASS: __________ DATE: ________

Solve each problem.

Example:
John studies 1 hr. 15 min. each night. How much time does he spend studying in 5 days?

1 hr. 15 min.
x5

5 hr. 75 min. =
1 hr. 15 min.
6 hr. 15 min.

1. Mr. Smith needs 2 pieces of pipe to fix the leaky sink. One piece is 2 ft. 8 in. and the other is 3 ft. 7 in. How much pipe does he need altogether?

2. Jean is 4 ft. 9 in. tall. Her mother is 5 ft. 6 in. tall. How much taller is Joan than her mother?

3. Mary is making 2 cakes. Each cake needs 2 cups, 6 oz. of flour. How much flour will she need altogether?

4. If we have 5 qts 12 oz. of juice and want to give equal amounts to 4 lunch tables, how much will each table receive?

5. John is a stock boy in the food store. He works 3 hr. 30 min each day. How long will he work in 5 days?

6. The school wants to put tile edging on both sides of the corrider. Each side is 45 ft. 9 in. How much edging will be needed?

7. Jason is making a model plane. He has 1 ft. 9 in. of wood. He must make 7 equal strips of wood. How long will each strip be?

Unit 26

7.10.03 METRIC UNITS

(1 lesson)

PERFORMANCE OBJECTIVES

The student will be able to...

7.10.03.01A select the appropriate unit of linear, volume and weight measure; e.g.,

Meter is linear measure.
Gram is a measure of weight.
Liter is a measure of volume.

7.10.03.02A describe the relationship between/among metric prefixes using base ten and identify the symbols for metric units. e.g.,

km,	hm,	dkm,	m,	dm,	cm,	mm
(1000)	(100)	(10)	(1)	(.1)	(.01)	(.001)

7.10.03

VOCABULARY

TERMS	SYMBOLS
meter	m
gram	g
liter	ℓ
kilo	
hecto	
deka	
deci	
milli	

GETTING STARTED

Write the following chart on the chalkboard.

DOLLARS

THOUSAND	HUNDRED	TEN	ONE	TENTHS	HUNDREDTHS

Ask a student to write the value, using numbers, for each column.
e.g.,

1000 100 10 1 .1 .01

Ask another student to give the name for one tenth of a dollar (dime) and one hundredth of a dollar (penny).

Say, "If I have $1.00 in pennies, how many pennies would I have? If I have $100 in $10 bills, how many bills do I have? If I have $10.00 in dimes, how many dimes do I have?"

Ask the students to answer each question, and write a statement of equivalence for each answer.
e.g.,

100 pennies = 1 dollar
10 ten dollar bills = $100
1000 dimes = $10

Repeat this procedure by asking other questions.

GETTING STARTED cont'd

Prepare and display three pictures, e.g., a highway, a container of milk and a group of apples. Ask the students what they would need to measure each item. Elicit a ruler, a measuring cup and a scale. Ask the students to name the appropriate measurement term used for each item. Elicit length, volume, capacity.

Write the following headings on the chalkboard.

Length Volume Weight

Elicit items which would go under each heading. Ask the students to write other items under each heading in their notebooks. Ask the students to exchange lists to compare.

7.10.03

7.10.03.01 **select the appropriate unit of linear, volume and weight measure**

STRATEGY 1: Prepare and display a meter stick, centimeter ruler, metric scales and an object, e.g., an apple.

Write these headings on the chalkboard.

Unit of Length	Unit of Weight	Unit of Liquid Capacity
Meter	Gram	Liter

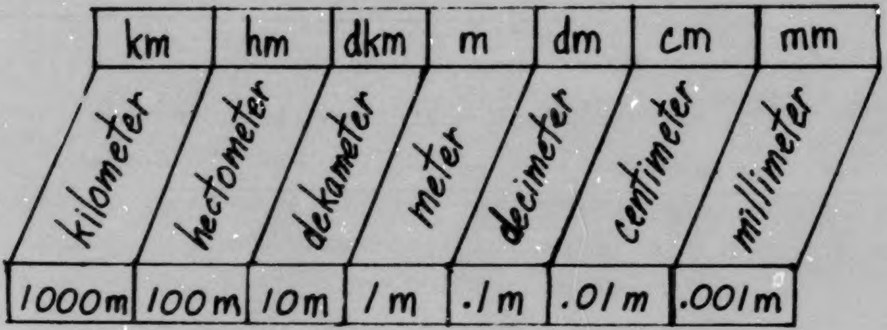
Ask the class what measurement system uses the above units. (metric) Ask the students to select the appropriate unit to measure an apple. (weight or gram) Write apple under Unit of Weight. Repeat this procedure for other items, e.g., a highway, juice in a bottle, a bag of onions.

STRATEGY 2: Prepare a meter stick, 1 liter measuring cup, gram balance scale and metric measuring tape. Say, "This is a meter stick. Use the meter stick to mark off one meter on the chalkboard. Ask several students to measure the length of the chalkboard with the metric tape. Say that we use meters to measure length. Show the balance scale and measure. Say that we use it to weigh in grams. Ask the students to weigh different objects on the scale, e.g., pencil, eraser, chalk, a paper cup... Display the graduated liter cup. Say that we measure volume with it. Ask several students to come up and fill the liter container with water to various marked levels, e.g., 750 ml., 250 ml., 500 ml. Display several objects and ask the students to select the appropriate measure for each, e.g., a bag of oranges, a container of juice, the height of a door.

7.10.03

7.10.03.02 **describe the relationship between/among metric prefixes using base ten and identify the symbols for metric units**

STRATEGY 1: Write the following chart on the chalkboard. e.g.,



km	hm	dkm	m	dm	cm	mm
kilometer	hectometer	dekameter	meter	decimeter	centimeter	millimeter
1000m	100m	10m	1m	.1m	.01m	.001m

Say, "Does this chart look familiar? (It is similar to a place value chart.) The metric system works on a base ten system. The meter is considered to equal one and the other units are fractions or multiples of it."

Display a meter stick. Say, "This is what one meter looks like. It is divided into ten decimeters, 100 centimeters and 1,000 millimeters." Show the markings on the ruler. Say, "A dekameter is ten meter sticks laid end to end, a hectometer is one hundred meters and a kilometer is one thousand meters."

STRATEGY 2: Prepare and distribute meter tapes to each student. Say, "This is a meter. It is divided into ten equal units called decimeters. (Point to divisions.) The decimeter is divided into ten equal units called centimeters. There are one hundred centimeters in a meter. The centimeter is divided

STRATEGY 2: cont'd

into ten equal units called millimeters. There are 1,000 millimeters in a meter." (10 X 100 = 1,000)

Write this chart on the chalkboard.

meter	decimeter	centimeter	millimeter
1m	.1m	.01m	.001m

Say, "There are also larger units in the Metric System. Imagine ten meter sticks lying end to end. This would be a dekameter. One hundred meters lying end to end would be one hectometer and one thousand meters would be a kilometer." Add these to the chart.

e.g.,

kilometer	hectometer	dekameter	meter	decimeter	millimeter
1,000m	100m	10m	.1m	.01m	.001m

Say, "These prefixes and their values are also used for grams and liters."

P.O. 7.10.03.01-.02

NAME: ______________________________ CLASS: __________ DATE: ________

A) Complete each statement using kilometer (km), meter (m), centimeter (cm), or millimeter (mm).

1. The length of a baseball bat is 94 _____.
2. The length of a walking step might be 65 _____.
3. My science book is 3 _____ thick.
4. The diameter of a quarter is 23 _____.
5. The distance from New York to San Francisco is about 1863 _____.

B) Give the weights of the following. Use milligram (mg), gram (g) or kilogram (kg).

1. A sack of potatoes: 3 _____
2. A pen: 30 _____
3. A raisin: 780 _____
4. A new born baby: 3½ _____
5. A can of soup: 312 _____

C) Complete.

1. 32 cm = ________ mm
2. 400 cm = ________ m
3. 26 m = ________ cm
4. 8 km = ________ m
5. 56 m = ________ mm
6. 6000 g = ________ kg
7. 9 kg = ________ gm
8. 500 g = ________ kg
9. 1500 mg = ________ gm
10. .08 gm = ________ mg
11. 8 L = ________ mℓ
12. 400 mℓ = ________ ℓ
13. 16 L = ________ mℓ
14. 12,000 mℓ = ________ ℓ
15. 530 mℓ = ________ ℓ

Unit 27

7.07.04 MULTIPLICATION OF DECIMALS

(3 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

7.07.04.01A determine and place the decimal point in the product of a given multiplication example if the digits of the product are given;

e.g.,

```
  4.236       3 places       4.236
x 2.804     + 3 places      x2.804
            ----------    11.877744
              6 places
```

7.07.04.02A solve verbal problems related to related to skills learned in this module.

e.g.,

Bob's Print Shop charges $.27 per page for each copy.

What would the cost of 74 copies be?

```
    .27
   x 74
   ----
    108
   189
 ------
 $19.98  for 74 copies
```

7.07.04

VOCABULARY

TERMS	SYMBOLS
decimal point standard numeral exponent base power	

GETTING STARTED

Write $\begin{array}{r}4.8\\ \underline{\text{x } 9}\end{array}$ on the chalkboard. Ask the students how many decimal places will be in the product. (one) Ask a student to come to the chalkboard to solve the example while the students solve the problem in their notebooks. (4.32)

Say, "How is this product different from the product of $\begin{array}{r}48\\ \underline{\text{x}9}\end{array}$? (The digits are the same, but the face value of the numbers is different, e.g., 43.2 as opposed to 432.) How is multiplication of decimals the same as multiplication of whole numbers? How is it different?"

Ask the students to solve the following examples.

$$\begin{array}{r}34.7\\ \underline{\text{x } 8}\end{array} \qquad \begin{array}{r}32.9\\ \underline{\text{x } 19}\end{array} \qquad \begin{array}{r}.836\\ \underline{\text{x } 3}\end{array} \qquad \begin{array}{r}16.3\\ \underline{\text{x } 25}\end{array}$$

Write $\begin{array}{r}48\\ \underline{\text{x}10}\end{array}$ on the chalkboard. Ask a student to come to the chalkboard to solve the example. Repeat with $\begin{array}{r}48\\ \underline{\text{x}100}\end{array}$ $\begin{array}{r}48\\ \underline{\text{x}1000}\end{array}$ $\begin{array}{r}48.\\ \underline{\text{x}10{,}000}\end{array}$

Say, "How are the products alike? How are they different? What is the relationship between the multiplier and the product?" (The multiplier and the product have the same number of zeros.)

Repeat the procedure using the following examples.

e.g.,

$$\begin{array}{r}537\\ \underline{\text{x}10}\end{array} \qquad \begin{array}{r}537\\ \underline{\text{x}100}\end{array} \qquad \begin{array}{r}537\\ \underline{\text{x}1{,}000}\end{array} \qquad \begin{array}{r}537\\ \underline{\text{x}10{,}000}\end{array}$$

7.07.04

7.07.04.01 **determine and place the decimal point in the product of a given multiplication example if the digits of the product are given**

STRATEGY 1: Write on the chalkboard.

e.g.,
$$\begin{array}{r} .32 \\ \underline{\text{x}\ .3} \end{array}$$

Say, "How do we solve this problem? (Multiply as if they were whole numbers.) We use the number of decimal places in each number of the example to determine the number of decimal places in the product. How many decimal places are there in .3? ...in .32? How many decimal places must we provide in the product numeral?"

e.g.,
$$\begin{array}{rl} .32 & \text{2 places} \\ \underline{\text{x}\ .3} & \underline{+\text{1 place}} \\ .096 & \text{3 places} \end{array}$$

Repeat the procedure using other examples.

e.g.,
$$\begin{array}{r} .24 \\ \underline{\text{x}.15} \end{array} \qquad \begin{array}{r} .1234 \\ \underline{\text{x}\ .06} \end{array}$$

STRATEGY 2: Write 6.5 x 3.5 = ☐ on the chalkboard. Say, "Let's change these decimals to fractions. What would they be?" ($6\frac{5}{10}$ x $3\frac{5}{10}$) Ask the students to solve the example.

e.g.,
$$6\frac{5}{10} \times 3\frac{5}{10} =$$

$$\frac{65}{10} \times \frac{35}{10} = \frac{2275}{100} = 22\frac{75}{100}$$

Say, "If we change $22\frac{75}{100}$ into a decimal, what would we get? ($22\frac{75}{100}$ = 22.75) If we do this each time, it would take us a long time, so we use another

7.07.04

STRATEGY 2: cont'd

method. First we multiply the numbers as if they were whole numbers. Then we count the number of places in the multiplicand and multiplier. We add the number of places. Starting from the right of the product, we count off the total number of decimal places."

e.g.,

```
   6.5    (1 place)
  x3.5    (1 place)
  32 5
 195
 22.75    (2 places)
```

7.07.04.02 **solve verbal problems related to skills learned in this module**

STRATEGY 1: Write the following problem on the chalkboard.

If the cost on the pricing label of a 4.25 lb. package of hamburger meat is listed as $9.45 and the price per pound is $1.89, determine whether the cost of the labeled package is accurate.

Say, "What are the steps that we must follow in solving a verbal problem?" Elicit the following steps.

1) Read the problem carefully.
2) Study the information to discover the given facts and decide what is to be found.
3) Decide upon the operation(s) you will use in solving the problem.
4) Estimate the answer by using rounded numbers.
5) Do the computations necessary to find the answer.
6) Check your answer.

Say, "What are the given facts? (The weight of the

STRATEGY 1: cont'd

package, the price per pound and the cost of the meat on the label) What do we want to find? (The actual cost and a comparison to the label cost) What operations need to be performed? (multiplication) What did you estimate the cost of the meat to be? (4 x 2 = $8). How does our estimate compare to the price on the label? (The cost of $9.45 appears to be incorrect. Find the actual cost of the package.

e.g.,

```
     $4.25
     x1.89
     38 25
    340 0
    425
   $8.0325
```

Repeat this procedure using other problems.

STRATEGY 2:

Write the following on problem on the chalkboard.

"Rock salt sells for $.18 per pound. What is the cost of 7.5 pounds?

Say, "What are the steps for solving verbal problems?"

e.g.,

What is to be found?
What number facts are given?
What "word clue(s)" is(are) given?
What operation(s) need(s) to be performed?
What is an estimate of the answer?
What is the actual answer?

Ask the students to follow the steps in solving the problem.

Repeat this procedure using other problems.

P.O. 7.07.04.01

NAME: ________________ CLASS: ________ DATE: ________

Find each product.
Put the decimal point in the correct place.

1. .7 x .6 = .42
2. .36 x .4
3. .56 x .9
4. .36 x 7
5. 463 x .09
6. .402 x .8
7. 3.62 x 3
8. 3.47 x 5
9. .024 x .2
10. 6.4 x 3
11. .056 x .3
12. 4.6 x 2.3
13. .46 x 2.3
14. .46 x .23
15. .046 x .23
16. .123 x 6
17. .0123 x .6
18. .0123 x .06
19. 34.5 x .012
20. 34.5 x .12

P.O. 7.07.04.02

NAME: ______________________ **CLASS:** ________ **DATE:** ________

Solve the following problems. Estimate your answer first.

1. A company sold 10,000 pencils at $.04 each. How much did it receive?

2. If, on the average, it costs a hotel $6.75 to feed a person for one day, how much will it cost to feed 100 people for a day?

3. What is the cost of 10.5 gallons of gasoline at $1.29 per gallon?

4. A steak costs $5.65/kg. How much will 0.8 kg cost?

5. A wholesaler charges $.175 for a cupcake. What is the cost of 10 cupcakes?

7.07.05 DIVISION OF DECIMALS

(3 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

*7.07.05.01A compute the quotient by correctly placing the decimal point given a division example in which the divisor is a power of 10; e.g.,

```
        .002469
1000)2.469000
```

7.07.05.02A compute the quotient to a specified decimal place given a division example in which the divisor is a whole number or a decimal; e.g.,

Compute the quotient to the nearest tenth.

```
                        19.59
1.4)27.430          14)274.30
                       -14
                        134
                       -126
                          8 3
                        - 7 0
                          1 30
                        - 1 26
                             4
```

19.59 rounded to the nearest tenth is 19.6

7.07.05.04A solve verbal problems related to skills learned in this module.

e.g.,

A butcher is selling a 12.5 lb. side of beef for $24.57.

How much does the beef cost per pound?

```
           1.965
      ___________
12.5)24.5.700
    -12 5
     ______
     12 0 7
    -11 2 5
     _______
        8 20
        7 56
     ________
          640
          625
          ____
           15
```

The cost per pound is $1.96.

GETTING STARTED

Write the following on the chalkboard.

Round each number to the nearest place indicated.

Decimal	whole	tenth	hundredth	thousandth
92.39948				

Say, "Between which two whole numbers does 92.39948 lie? (92 and 93) To which number is it closer? (92) Why?" (We use the tenths place to determine rounding to a whole number and 92.3 is less than 92.5.)

Repeat this procedure using 63.21456 and 3.59528.

Say, "Suppose a grand prize of $3456 is to be shared by ten people. How much does each person win?" (The prize for one person is found by dividing the grand prize by the number of people sharing it.) Ask a student to do the division on the chalkboard.

e.g.,

```
          345 6/10
   10 ) 3456
        30
         45
         40
          56
          50
           6
```

Ask the student for his/her result in dollars and cents. ($\$345\frac{6}{10}$ is the same as $345.60.)

Say, "Suppose the grand prize of $3456 has to be shared by 100 people, now how much will each person win?" Repeat this procedure with 1000 winners.

Ask the students if there is another way to get to $345.60, $34.56 or $3.46 without having to actually divide. (We can move the decimal place to the left for each power of ten.)

7.07.05

7.07.05.01 **compute the quotient by correctly placing the decimal point given a division example in which the divisor is a power of 10**

STRATEGY 1: Write the following on the chalkboard.

e.g.,

$3456 \div 10 = 345.6$
$3456 \div 100 = 34.56$
$3456 \div 1000 = 3.456$
$3456 \div 10{,}000 = .3456$

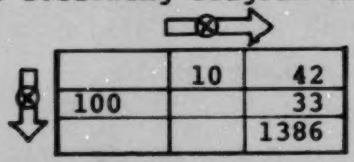
Say, "Can you find any pattern in the quotients? There is a shortcut you can learn whenever you need to divide a problem that has a power of ten for a divisor. To divide a number by ten, simply move the decimal point in the dividend one place to the left." Ask a student to identify the dividend in the first example. Ask another student where to place the decimal point in 3456 so as not to change the number. Write on the chalkboard 3456. ÷ 10 = 345.6. Go over the shortcut for division by 100 or 10^2 (move the decimal point two places to the left), division by 1000 or 10^3 (move the decimal point three places to the left) and division by 10,000 or 10^4 (move the decimal point four places to the left).

Divide the class into two teams. A member of Team I writes a division example on the chalkboard so that the dividend is a decimal and the divisor is a power of ten. A Team II member solves the problem at the chalkboard using the shortcut. Should a Team II member be unable to solve the problem, a Team I member may be called upon. If the problem is done correctly by a Team II member, another Team II member writes a similar problem for Team I to solve. If the problem is solved on the first try, award that team 10 points. Add ten points to the award each time the problem has to be passed on to the other team.

7.07.05

STRATEGY 2: Draw the following diagram on the chalkboard.

e.g.,



	10	42
100		33
		1386

Say, "The empty boxes should be filled in with factors." Remind students that a missing factor can be found by dividing the product by the given. Say, "If the boxes are filled in correctly, the two factors in the third row when multiplied should equal 1386. The third row members are products of their respective columns." Ask several students to fill in the missing factors.

Say, "Who remembers the shortcut for multiplication by 10? (We move the decimal point one place to the right.) What happens to the decimal point in dividing a number by ten?"

Repeat the remaining facts of the chart similarly.

e.g.,

100 x .33 = 33 4.2 x 100 = 420 10 x .33 = 3.3

33 ÷ 100 = .33 420 ÷ 100 = 4.2 3.3 ÷ 10 = .33

Repeat this procedure using other examples.

7.07.05.02 **compute the quotient to a specified decimal place given a division example in which the division is a whole number or a decimal**

STRATEGY 1: Say, "In July, 1983, Ron Cey of the Chicago Cubs had been at bat 290 times and had 75 hits. Who knows how to determine his batting average?" (The number of hits made is divided by the number of times at bat. Note that the average is always written to the nearest thousandth.)

7.07.05

STRATEGY 1: cont'd

e.g.,

```
        .258
290 )75.000
    -58 0
     17 00
    -14 50
      2 500
     -2 320
        180
```

Ask the class if .258 is the correct batting average. (To find the average to the nearest thousandth, you must carry out the division to the ten thousandths place.) Ask a student to come to the chalkboard and complete the division (.2586) and give the quotient to the nearest thousandth. Say, "If it is necessary to give the answer in tenths, to what place must you carry out the division?" Repeat this question for hundredths.

STRATEGY 2: Say, "Suppose you are sent to the supermarket to buy detergent. The super giant economy box weighing 10 lbs. sells for $6.79. The regular standard size weighing 4.5 lbs. sells for $2.75. Which gives you more for your money?"

e.g.,

```
         .611              .679
4.5 )2.7500         10 )6.790
      2 70               6 0
         50               79
         45               70
          50               90
```

Say, "To determine the cost per pound, you would have to round off each answer to the nearest hundredth. The 4.5 pound box would cost $.61 per pound and the giant economy box would cost $.68 per pound. Which is a better buy?" ($.61 per pound)

Repeat this procedure using other examples.

7.07.05.03 **solve verbal problems related to skills learned in this module**

7.07.05

STRATEGY 1: Write the following on the chalkboard.

> A wooden board, 17.45 meters long, has to be cut into 5 equal pieces. How long will each piece be to the nearest tenth?

Suggest replacing 17.45m with a simpler number, e.g., 20m. Say, "Now we have a 20 meter board to be cut into 5 equal pieces." Ask a student to solve the simpler problem, e.g., 20m ÷ 5 = 4m. Say, "Now let's use the original numbers." Ask a student come to the chalkboard to solve the problem.

e.g.,

```
     3.49m
5 )17.45m
```

Ask a student to do the rounding. (3.5m) Repeat this process using the other problems.

STRATEGY 2: Write the following on the chalkboard.

Nutrigrain	12 oz./$1.49
Uncle Sam's	10 oz./$ .95

Say, "Both breakfast cereals contain approximately the same nutriments. Which one is the better buy?" Say that it is necessary to compare the cost per ounce of each cereal by dividing the cost of the cereal by its weight. Ask the students to find the quotients to the nearest cent (hundredths). Suggest that the students find the cost of Uncle Sam's by the shortcut method.

e.g.

```
     Nutrigrain                  Uncle Sam's

      $ .124 → $.12          .95 ÷ 10 = $.095
 12 )$1.490                            $.09
      1 2                              +.01
        29                             $.10
        24
         50
         48
          2
```

Elicit from the class which is the better buy (Uncle Sam's). Repeat this procedure using other problems.

P.O. 7.07.05.01

NAME:____ **CLASS:**__________ **DATE:**________

Compute the quotient by division or by the short-cut method.

1. $10\,\overline{)\,54}$	2. $10\,\overline{)\,5.4}$	3. $.54 \div 10 =$	4. $\frac{5.4}{100} =$
5. $100\,\overline{)\,540}$	6. $540 \div 1000 =$	7. $\$5.40 \div 1000 =$	8. $\frac{.54}{100} =$
9. $1000\,\overline{)\,38.62}$	10. $100\,\overline{)\,246.8}$	11. $.068 \div 10 =$	12. $\frac{685}{10} =$
13. $.7 \div 1000 =$	14. $100\,\overline{)\,1}$	15. $603 \div 100 =$	16. $100\,\overline{)\,305.2}$

P.O. 7.07.05.02

NAME: ____________________ CLASS: ____ DATE: ________

Round each quotient to the nearest whole number, nearest tenth and nearest hundredth.

	Nearest Whole	Nearest Tenth	Nearest Hundredth
① 78.6 ÷ 23 =	3	3.4	3.42
② 21.75 ÷ 9 =			
③ 4.4 ÷ .06 =			
④ .8 ÷ 5.8 =			
⑤ 268 ÷ .16 =			
⑥ 5.273 ÷ .286 =			
⑦ 1.35 ÷ 1.06 =			
⑧ 4.82 ÷ .68 =			
⑨ 75 ÷ .36 =			
⑩ 709.64 ÷ .215 =			

P.O. 7.07.05.03

NAME:______________________________ CLASS:________ DATE:________

Solve each word problem.

1. Mr. Martin paid $4.32 for 2.5 lb. of hamburger meat. What is the cost of one pound of hamburger to the nearest cent?

2. A new pencil is 18.8 cm long. If Charlene uses 0.5 cm of the pencil by writing each day, how many days will the pencil last?

3. A group of students collected $9.99 for 20 pounds of candy. How much can they spend on one pound of candy to the nearest cent?

4. A car gets 28 miles to one gallon of gas. If it was driven 802 miles, how many gallons did it use?

5. What is the better buy, a 2 pound bag of potatoes for $.79 or a 5 pound bag of potatoes for $1.89?

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Unit 29

7.12.01 PROBABILITY

(2 lessons)

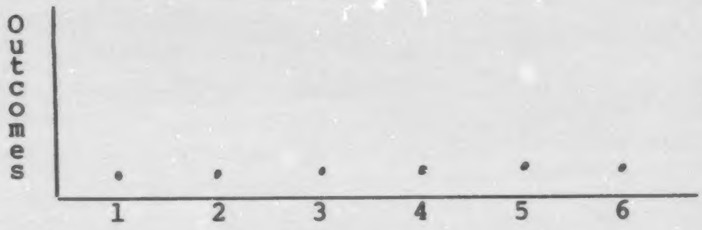
PERFORMANCE OBJECTIVES

The student will be able to...

*7.12.01.01A construct a sample space for various experiments;

e.g.,

Tossing a die



Possible Meals

	Deserts	
	Cake (C)	Ice Cream (I)
Tuna (T)	TC	TI
Salami (S)	SC	SI
Peanut Butter (P)	PC	PI

*7.12.01.02A read and use the expression P(A) to represent the probability of event A;

e.g.,

A = the light will be on

$P(A) = \frac{1}{2}$

7.12.01.03A write the probabilities involved with simple experiments: flipping a coin, rolling a die. e.g.,

B = a five will be picked from a deck of cards

$P(B) = \frac{4}{52} = \frac{1}{13}$

7.12.02

VOCABULARY

TERMS

probability
event
outcome
favorable outcome
sample space

GETTING STARTED

Say, "A basketball player made 8 baskets out of 10 shots. What is the ratio of baskets to shots? ($\frac{8}{10}$ or $\frac{4}{5}$) What is the ratio of shots to baskets?" ($\frac{10}{8}$ or $\frac{5}{4}$)

Repeat this procedure using other examples.

Prepare and display a set of 3 blocks where two blocks are red and one block is green. Say, "What is the ratio of red blocks to the total number of blocks? ($\frac{2}{3}$) What is the ratio of green blocks to the total number of blocks?" ($\frac{1}{3}$)

Prepare and display a set of playing cards. Ask a student to pick a card, e.g., ace of spades. Say, "What is the ratio of the ace of spades to the total number of playing cards in the deck?" ($\frac{1}{52}$)

Repeat this procedure with other experiments of picking cards.

7.12.01

7.12.01.01 **construct a sample space for various experiments**

STRATEGY 1: Prepare and display three cards that show the letters X, Y and Z. Say, "What are all of the possible arrangements of the three cards?" List the six arrangements on the chalkboard.

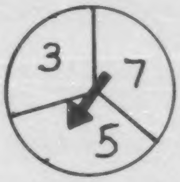
e.g.,

XYZ, XZY
YXZ, YZX
ZXY, ZYX

Say, "This is a sample space. A sample space shows all possible outcomes in an experiment. A sample space may consist of a list, a table or a diagram." Repeat this procedure using other examples.

STRATEGY 2: Prepare and display a spinner with the numbers 3, 5 and 7.

e.g.,



Say, "The spinner is spun two times. What are all of the possible outcomes?" Emphasize that a specific order should be followed in listing all the possible outcomes by saying, "List all possible outcomes beginning with 3. List all the possible outcomes beginning with 5. List all the possible outcomes beginning with 7."

e.g.,

Possible Outcomes

Beginning with 3 - (3,3) (3,5) (3,7)
Beginning with 5 - (5,5) (5,7) (5,3)
Beginning with 7 - (7,7) (7,3) (7,5)

Say, "This is a sample space which shows all outcomes in an experiment. A sample space may consist of a list, a table or a diagram."

Repeat this procedure using other examples.

7.12.01

7.12.01.02 **read and use the expression P(A) to represent the probability of event A**

STRATEGY 1: Say, "The probability of flipping a coin one time and getting tails is $\frac{1}{2}$. The mathematical notation used to indicate this is $P(T) = \frac{1}{2}$. T indicates the event that a tail will occur. $P(T) = \frac{1}{2}$ is read as the probability of event T is $\frac{1}{2}$." Ask how we indicate in this notation that the probability of heads is $\frac{1}{2}$. ($P(H) = \frac{1}{2}$ where H equals heads.) Repeat this procedure with other examples.

STRATEGY 2: Prepare and display three cards that show the letters M, C and M. Place the cards in a bag. Write $P(M) = \frac{2}{3}$ and $P(C) = \frac{1}{3}$ on the chalkboard. Say, "What do $P(M) = \frac{2}{3}$ and $P(C) = \frac{1}{3}$ indicate?" Elicit that $P(M) = \frac{2}{3}$ indicates that the probability of selecting a card with an M on it is 2 out of 3 ($\frac{2}{3}$). $P(C) = \frac{1}{3}$ indicates that the probability of selecting a card with a C on it is 1 out of 3. The selection of M or C are particular events. Repeat this procedure using other examples.

7.12.01.03 **write the probabilites involved with simple experiments: flipping a coin, rolling a die**

STRATEGY 1: Prepare and display a bowl containing four green marbles and three black marbles. Say, "What is the probability that a marble picked without looking will be black?" Write P(black) = ? on the chalkboard. Say that the probability of picking a black marble is $\frac{3}{7}$ and this may be shown as follows.

7.12.01

STRATEGY 1: cont'd

e.g.,

$$P(\text{an event}) = \frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}$$

Say, "What is the probability that a marble picked without looking will be green? ($\frac{4}{7}$) What is the probability that any marble will be picked? (1) If an event is certain to happen, the probability of the event is 1. What is the probability that a red marble will be picked? (0) If an event cannot happen or never happens, the probability of the event is 0."

Repeat this procedure using other examples.

STRATEGY 2: Place four pennies, three nickels and six dimes into a cup. Say, "The cup is to be shaken until one coin falls out. What is the probability that the first coin to fall out will be a penny? ($\frac{4}{13}$) The probability of an event is equal to the ratio of the number of favorable outcomes (outcomes of events we are interested in) to the total number of possible outcomes for an event." This may be written on the chalkboard as follows.

e.g.,

$$P(\text{an event}) = \frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}$$

Say, "What is the probability that the first coin to fall out of the cup will be a quarter? (0) If an event cannot happen or never happens, the probability of the event is 0. What is the probability that the first coin to fall out of the cup will be a U. S. coin? (1) If an event is certain to happen, the probability of the event is 1."

Repeat this procedure using other examples.

P.O. 7.12.01.01

NAME: ______________________ CLASS: ________ DATE: ________

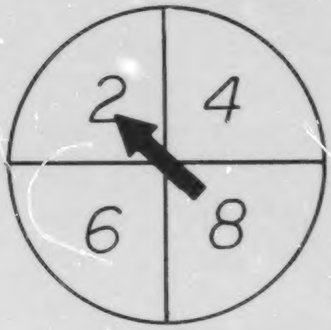
I. List all possible arrangements.
II. List all possible outcomes.

I

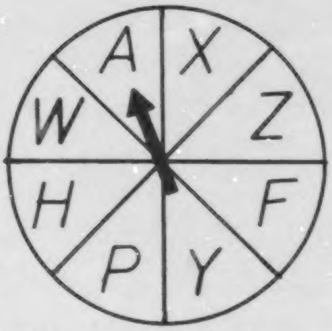
C A T

F O U R

II



Two Spins



One Spin

P.O. 7.12.01.02

NAME: ______________________ CLASS: __________ DATE: ________

I. Write in proper mathematical notation.

1) R = selecting a red marble from a box.
The probability of selecting a red marble from a box is 3 out of 7.

P(R) = $\frac{3}{7}$

2) B = selecting a blue marble from a box.
The probability of selecting a blue marble
from a box is 1 out of 7.

____

3) G = selecting a green marble from a box.
The probability of selecting a green marble
out of a box is 2 out of 7.

____

II. Write a sentence.

4) P(A) = $\frac{2}{5}$ The probability of event A is $\frac{2}{5}$

5) P(C) = $\frac{1}{7}$ ______________________

6) P(W) = 1 ______________________

7) P(R) = 0 ______________________

P.O. 7.12.01.03

NAME: ____________________ CLASS: ________ DATE: ________

Slips of paper with the following numbers are put into a box:

10 10 10 9 9 8 8 8 8 7 6 5 5 4

If one slip of paper is drawn, what is the probability that it will be:

(1) P(10) = $\frac{3}{14}$

(2) P(9) =

(3) P(8) =

(4) P(7) =

(5) P(6) =

(6) P(5) =

(7) P(4) =

(8) P(3) =

(9) P(even number) =

(10) P(odd number) =

(11) P(a number $<$ 9) =

(12) P(a number) =

(13) P(a number $>$ 9) =

(14) P(11) =

(15) P(a number $<$ 5) =

Unit 30

7.12.02 PROBABILITY WITH A SPINNER

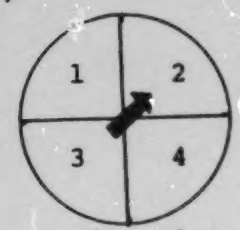
(2 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

7.12.02.01A state that the sum of the probabilities in a given experiment is always 1;

e.g.,



(in spinner above)

$P(1) = \frac{1}{4}$

$P(2) = \frac{1}{4}$

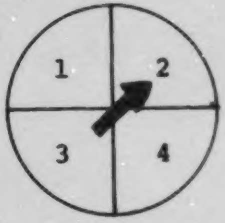
$P(3) = \frac{1}{4}$

$P(4) = \frac{1}{4}$

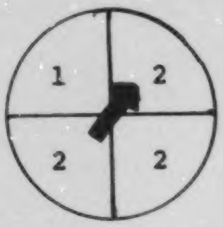
$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{4}{4} = 1$

*7.12.02.03A state and use the meaning of equally likely events;

e.g.,



Spinner A



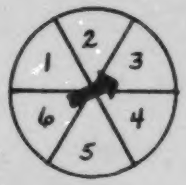
Spinner B

The likelihood that 2 will come up on Spinner A is equal to the likelihood that 4 will come up.

The likelihood that 2 will come up on Spinner B is unequal to the likelihood that 1 will come up.

7.12.02.03A state and use the meaning of complementary events to their probabilities.

e.g.,



$P(3) = \frac{2}{6} = \frac{1}{3}$ $P(\text{not } 3) = \frac{4}{6} = \frac{2}{3}$

complement

P (not an event) = 1-P(event)

66

7.12.01

VOCABULARY

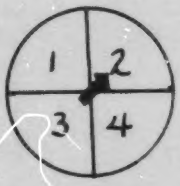
TERMS

event
outcome
likelihood
complimentary
complement

GETTING STARTED

Prepare and display a spinner with four numbers on it. All selections should be of equal size.

e.g.,



Say, "What are some possible outcomes, if we spin the spinner once?" List the outcomes on the chalkboard, e.g., 1, 2, 3, 4. Say, "What are some possible outcomes if we spin the spinner twice? A specific method could be used to list the total possible outcomes."

e.g.,

(1,1)	(1,2)	(1,3)	(1,4)
(2,1)	(2,2)	(2,3)	(2,4)
(3,1)	(3,2)	(3,3)	(3,4)
(4,1)	(4,2)	(4,3)	(4,4)

Actually spin the spinner and record the results for one spin and then for two spins.

Prepare and display a die to the students. Say, "What are some possible outcomes if the die is rolled once? (1, 2, 3, 4, 5, 6.) What is the probability that a 3 will come up if the die is rolled once? ($\frac{1}{6}$) The probability of an event is equal to the number of favorable outcomes divided by the total number of possible outcomes." Write the following on the chalkboard.

$$\text{P (an event)} = \frac{\text{the number of favorable outcomes}}{\text{the total number of possible outcomes}}$$

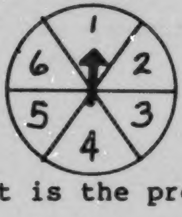
Say, "What is the probability that a seven will come up if the die is rolled once? (0) If an event cannot happen or never happens, the probability of the event is zero. Repeat this procedure with other examples.

7.12.02

7.12.02.01 state that the sum of the probabilities in a given experiment is always 1

STRATEGY 1: Prepare and display a spinner with six numbers where all sections are equal size.

e.g.,



Say, "What is the probability that a one will come up? two?...

e.g.,

$P(1) = \frac{1}{6}$ $\quad P(4) = \frac{1}{6}$

$P(2) = \frac{1}{6}$ $\quad P(5) = \frac{1}{6}$

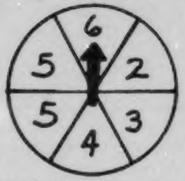
$P(3) = \frac{1}{6}$ $\quad P(6) = \frac{1}{6}$

What is the sum of the probabilities for this spinner? ($\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{6}{6} = 1$) In any experiment the sum of the probabilities of the possible events equals one."

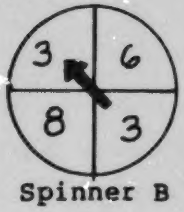
Repeat this procedure using other examples.

STRATEGY 2: Draw two spinners on the chalkboard.

e.g.,



Spinner A



Spinner B

Ask the students to complete the probability values shown for spinners A and B assuming one spin.

e.g.,

Spinner A	Spinner B
P(even number) = $\frac{3}{5}$	P(3) = $\frac{2}{4} = \frac{1}{2}$
P(odd number) = $\frac{2}{5}$	P(6) = $\frac{1}{4}$
	P(8) = $\frac{1}{4}$

Say, "What is the sum of the probabilities for

STRATEGY 2: cont'd

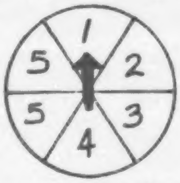
Spinner A? ($\frac{5}{5} = 1$) What is the sum of the probabilities for Spinner B? ($\frac{4}{4} = 1$) The sum of the probabilites in a given experiment is always one."

Repeat this procedure using other examples.

7.12.02.02 **state and use the meaning of equally likely events**

STRATEGY 1: Prepare and display the following spinner to the students where all selections are of equal size.

e.g.,



Ask several students to come to the chalkboard and complete one probability equation.

e.g.,

$P(1) = \frac{1}{6}$

$P(2) = \frac{1}{6}$

$P(3) = \frac{1}{6}$

$P(4) = \frac{1}{6}$

$P(5) = \frac{2}{6} = \frac{1}{3}$

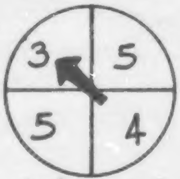
Say, "Is the likelihood that two will come up on the spinner equal to the likelihood that four will come up on the spinner. (Two and four have the same probability, $\frac{1}{6}$, of occurring.) Two events that have the same chance of occurring are called 'equally likely events'. Is the likelihood that two will come up on the spinner equal to the likelihood that five will come up on the spinner?" (Two and five are not equally likely events. Five has a greater likelihood of occurring because it has a higher probability than that for two.)

Repeat this procedure with other examples.

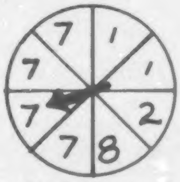
7.12.02

STRATEGY 2: Draw two spinners on the chalkboard.

e.g.,



Spinner A



Spinner B

Ask the students to list the probability values for Spinner A and Spinner B for one spin.

e.g.,

Spinner A	Spinner B
P(3) = $\frac{1}{4}$	P(1) = $\frac{2}{8} = \frac{1}{4}$
P(4) = $\frac{1}{4}$	P(2) = $\frac{1}{8}$
P(5) = $\frac{2}{4} = \frac{1}{2}$	P(7) = $\frac{4}{8} = \frac{1}{2}$
	P(8) = $\frac{1}{8}$

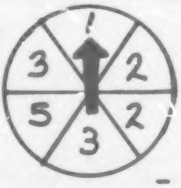
Say, "Which number in Spinner A is most likely to come up? (five) Which number is more likely to occur in Spinner A, 3 or 4? (The likelihood that 3 will come up is equal to the likelihood that 4 will come up since both numbers have the same probabilities.) Events with the same probabilities are called 'equally likely events'. Are there equally likely events in Spinner B?" (8 and 2)

Repeat this procedure using other examples.

7.12.02.03 **state and use the meaning of complementary events to their probabilities**

STRATEGY 1: Prepare and display a spinner with four numbers where all of the sections are of equal size.

e.g.,



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7.12.02

STRATEGY 1: cont'd

Say, "What is the probability of a 5 turning up on one spin? (P(5) = $\frac{1}{4}$) What does P(not 5) mean? P(not 5) is the probability that 5 will not occur on one spin.) P(not 5) = $\frac{3}{4}$ because 3 outcomes out of 4 possible outcomes are not 5. The events of getting a 5 or not getting a 5 are called complementary events. What is the sum of P(5) and P(not 5)? ($\frac{1}{4} + \frac{3}{4} = 1$) If A represents an event and P(A) + P(not A) = 1, what does P(not A) equal?" (P(not A) = 1 -P(A)) Ask the students to apply this rule for P(3), e.g., P(3) = $\frac{3}{6} = \frac{1}{2}$ and P(not3) = $1 - \frac{1}{2} = \frac{1}{2}$.

Repeat this procedure using other examples.

STRATEGY 2:

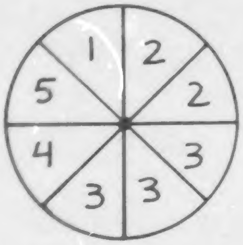
Prepare and display two red blocks or marbles and one blue block or marble. Place the items in a bag. Say, "What is the probability that we will reach into the bag and take out a red block (marble)? (P(Red) = $\frac{2}{3}$) What is the probability that you will not choose a red block (marble)? (P(not Red) = $\frac{1}{3}$ because the only other choice is the blue item). What do you notice about the P(Red) and P(not Red)? ($\frac{2}{3} + \frac{1}{3} = 1$) Another method of finding the compliment or the probability that something will not happen is to subtract the probability that it will happen from one, e.g., P(not Red) = $1 - \frac{2}{3} = \frac{1}{3}$."

Repeat this procedure using other items.

P.O. 7.12.02.01

NAME: ____________________ CLASS: ________ DATE: ________

Find the probability values for 1 spin of the spinner.
Add the sum of the probabilities to check your answer.



1. P (1) = ______________

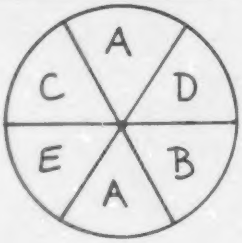
2. P (2) = ______________

3. P (3) = ______________

4. P (4) = ______________

5. P (5) = ______________

6. P (1) + P (2) + P (3) + P (4) + P (5) = ______________



7. P (A) = ______________

8. P (B) = ______________

9. P (C) = ______________

10. P (D) = ______________

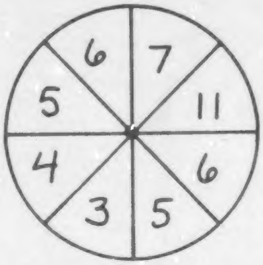
11. P (E) = ______________

12. P (A) + P (B) + P (C) + P (D) + P (E) = ______________

P.O. 7.12.02.02

NAME: ______________________________ CLASS: __________ DATE: ________

I. Write True or False (assume 1 spin).



1. The likelihood that 3 will come up on the spinner is equal to the likelihood that 4 will come up. TRUE

2. The likelihood that 5 will come up on the spinner is equal to the likelihood that 7 will come up. __________

3. The likelihood that an odd number will come up on the spinner is equal to the likelihood that an even number will come up. __________

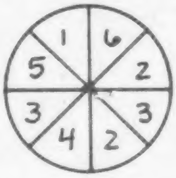
4. The likelihood that 5 will come up on the spinner is unequal to the likelihood that 6 will come up on the spinner. __________

5. The likelihood that 11 will come up on the spinner is unequal to the likelihood that 5 will come up. __________

6. The likelihood that numbers greater than 6 will come up on the spinner is equal to the likelihood that numbers less than 5 will come up. __________

7. The likelihood that 4 will come up on the spinner is unequal to the likelihood that 11 will come up. __________

II. List the probability values for 1 spin.
List the equally likely events.



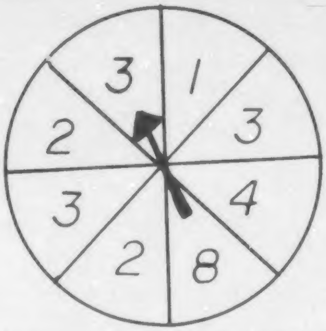
Equally likely events

8. P(1) =
9. P(2) =
10. P(3) =
11. P(4) =
12. P(5) =
13. P(6) =

P.O. 7.12.02.03

NAME: ______________________ CLASS: __________ DATE: ________

Find the probability value for one spin.



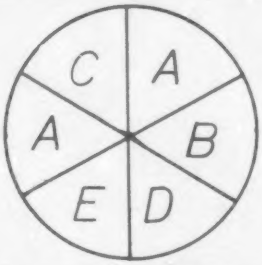
1. P(1) = P (not 1) =

2. P(2) = P (not 2) =

3. P(3) = P (not 3) =

4. P(4) = P (not 4) =

5. P(8) = P (not 8) =



6. P(A) = P (not A) =

7. P(B) = P (not B) =

8. P(C) = P (not C) =

9. P(D) = P (not D) =

10. P(E) = P (not E) =

Unit 31

7.11.03 VARIABLES

(3 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

*7.11.03.01A state the meaning of a variable; e.g.,

He is president of our club.
He holds the place of a name and is a variable.

7.11.03.02A determine the value of an equation. e.g.,

$n + 8 = \square$ when $n = 6$
$6 + 8 = 14$

7.11.03

VOCABULARY

TERMS	SYMBOLS
variables equation	□ , △, X, Y, Z

GETTING STARTED

Write the following on the chalkboard.

Which of the following can be judged true or false?

a) 8 + 3 = 11 b) 12 - 5 = 7 c) 3 x □ = 21

Say, "Any mathematical sentence that can be judged true or false is called a statement. Which of the above sentences are statements? (a,b) Any sentence that can not be judged true or false is called a phrase."

Write the following on the chalkboard. Say, "Which of the following are statements and which are phrases?"

1) 5 + 9 = 14
2) x is an even number.
3) She is wearing a blue dress.
4) y + 8 = 19
5) 2 + □ = 6

Write four times eight equals thirty-two on the chalkboard. Ask a student to translate the sentence into an equation, e.g., 4 x 8 = 32.

Repeat this procedure using other sentences.

7.11.03

7.11.03.01 **state the meaning of a variable**

STRATEGY 1: Write the following on the chalkboard.

$12 \times 9 = 108$, $16 - 5 = 8$, $x + 5 = 9$

Say, "Which of these can be judged true and/or false? Why can't $x + 5 = 9$ be judged true or false? (We don't know the value of x.) The symbol in the equation which holds the place for a number is called a variable."

Write the following on the chalkboard.

What is the variable in each of the following sentences?

a) $\square + 7 = 15$ c) $y - 8 = 15$
b) He has brown eyes. d) $x + 7 = 9$

Repeat this procedure using other examples.

STRATEGY 2: Say, "What mathematical symbols can be used to express the following?" Write "a number decreased by 3 is 9" on the chalkboard. Say, "What symbol can we use to show 'a number'? (x, n, m...) We call this symbol a variable since we can vary what it represents."

Repeat this procedure using other sentences.

7.11.03.02 **determine the value of an equation**

STRATEGY 1: Write $5 \times N + 6$ on the chalkboard. Say, "What is the value of this phrase? Why can't we determine its value? (We don't know what N represents.) Suppose we replace N with a three. What is the value?" Ask a student to come to the chalkboard to

STRATEGY 1:
cont'd

solve the equation.

e.g.,

$$5 \times N + 6 =$$
$$5 \times 3 + 6 =$$
$$15 + 6 =$$
$$21$$

Say, "5 x N + 6 is called a literal expression. A literal expression has a numerical value when the variable is replaced by a number, e.g., 21 is the numerical value of the literal expression 5 x N + 6 when N is replaced by 3." Elicit other values for N and repeat this procedure. Ask what will happen to the value of the literal expression 5 x N + 6 as N increases? As N decreases?

STRATEGY 2:

Write the following on the chalkboard.

> Which number(s) from zero to five will make the following sentence true?
> Four times a number decreased by three equals nine.

Ask a student to write the expression using numerical symbols. Say, "What can we substitute for the word number?" (a variable, e.g., 4 x n -3 = 9) Ask the students to replace "n" with zero. Say, "Does zero make the sentence true?" Repeat this procedure using the numbers one through five.

Write 13 - b on the chalkboard. Say, "The value of the equation will change depending on the value of b. What will the value be when b = 5? (13 - 5 = 8) when b = 7?" (13 - 7 = 6)

Repeat this procedure using other equations and values.

P.O. 7.11.03.01

NAME: ____________________ CLASS: __________ DATE: __________

I. Circle the variable in each of the following open sentences.

1. He is wearing brown slacks.

2. $n + 5 = 12$

3. $5a + 6 = 16$

4. $X = 8$

5. $\frac{1}{2}$ of $\square = 6$

II. Using the letter Y as a variable, write the following phrases.

1. Eight times a week.

2. A number plus thirty.

3. A number increased by twelve.

4. A number divided by 6.

5. A number diminished by eight.

P.O. 7.11.03.02

NAME:______________________________CLASS:__________DATE:________

I. Evaluate each expression when X = 10.

I.

1. $x - 7 =$
2. $5x =$
3. $x - 3 =$
4. $x - 125 =$
5. $x \div 2 =$
6. $x + 10 =$
7. $25 - x =$
8. $59 - x =$
9. $35 \cdot x =$
10. $x + 43 =$

II. Evaluate 35 - m for the given value of m.

11. $m = 6$
12. $m = 19$
13. $m = 27$
14. $m = 15$
15. $m = 9$

III. Evaluate 5 x a for the given value of a.

16. $a = 17$
17. $a = 49$
18. $a = 12$
19. $a = 25$
20. $a = 135$

7.11.04 FUNCTIONS (RELATIONSHIP OF VARIABLES)

(2 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

7.11.04.01B recognize a function in the form $y = ax$; e.g.,

Perimeter of a square = 4s
$P = 4s$

7.11.04.02B recognize a function in the form $y = ax + b$. e.g.,

The post office charges 20¢ for a letter which weighs 1 ounce. For every ounce over the first it charges 17¢ per ounce. If a letter weighs n ounces the charge is cost = 17(N-1)+20.

cost = 17(n-1) + 20
cost = 17n - 17 + 20
cost = 17n + 3

7.11.04

VOCABULARY

TERMS

variable
function
relationship

GETTING STARTED

Introduce the game "I'm Thinking of a Number". Say, "I am thinking of a number. When I add 3, the sum is 9. What is my number? (six) I am thinking of a number that when multiplied by 5 the product is 40. Of which number am I thinking?" (eight)

Form teams to play the game. Each team should think of a number for the other team to guess.

Write several open sentences on the chalkboard and ask the students to determine what operation is needed to make the sentence true.

e.g.,

$3 \square 4 = 12$ (multiplication)
$3 \square 4 = 7$ (addition)
$3 \square 4 \square 2 = 14$ (Both multiplication and addition)
$15 = 6 \square 3 \square 3$ (Both multiplication and subtraction)

Repeat this procedure using other examples.

7.11.04

7.11.04.01 **recognize a function in the form y = ax**

STRATEGY 1: Say, "If I had six dozen eggs, how many eggs would I have? (72) How did you reach the solution? (6x12 = 72) If I had three dozen eggs, how many would I have?" (36 since 3x12 = 36) Repeat this process with five dozen eggs. Ask the class to compare the three answers (6x12 = 72, 3x12 = 36, 5x12 = 60), and make some observations. (In each case we are multiplying numbers by 12, since there are 12 eggs to a dozen.) Ask the class for a statement or rule to find out how many eggs we would have if we had N dozen. Write on the chalkboard and say, "Let N equal the number of dozens and let E equal the number of eggs." Show on the chalkboard that

when we had 6 dozen eggs we had, 6 x 12 = 72;
when we had 3 dozen eggs we had, 3 x 12 = 36;
when we had 5 dozen eggs we had, 5 x 12 = 60;
now, we have N dozen eggs, so we have, N x 12 = E.

Say, "We now have a general equation by which we can find out how many eggs we have if we know the number of dozens. In mathematics, we have a special way of writing an equation. When we multiply a numeral by a letter, we can put them together without a multiplication sign (x)." Demonstrate by writing on the chalkboard, N x 12 = 12N. Rewrite the equation as 12N = E. Ask the students to write an equation to find the cost of N, 12 cent stamps. (12N = C)

Write the following equations on the chalkboard.

P = 4S (perimeter of a square = 4 x sides)
C = πD (circumference of a circle = πxdiameter)
A = bh (area of a parallelogram = base x height)

Elicit other formulas. Repeat this procedure using other examples.

7.11.04

STRATEGY 2: Write the following equation on the chalkboard, P = 4S. Say, "What does this equation mean? When a number is next to a letter, it means that the operation to be used is multiplication. This equation tells us that when we multiply 4 by a number (S), we will get our answer (P). If S = 4, then 4x4 = P = 16. If S = 5, then 4x5 = P = 20. As S changes, so does P. This formula is used to find the perimeter of a square where P = perimeter and S = side."

Repeat this procedure using other equations.

7.11.04.02 **recognize a function in the form y = ax + b**

STRATEGY 1: Present the following problem.

> A taxi gets \$1.00 for the first $\frac{1}{9}$ mile and \$.10 for each additional $\frac{1}{9}$ of a mile. How much would you have to pay for a 3 mile ride?

Say, "It costs \$1.00 for the first $\frac{1}{9}$ mile and \$.80 additional for the other $\frac{8}{9}$ of the mile. After that, each mile would be counted as $\frac{9}{9}$ or 9 x .10 = .90. Therefore the first whole mile would be \$1.80 and the second and third miles would be \$.90 each, totaling \$3.60."

Present the students with different mileages and ask them to calculate the answers. Ask the students for a formula to solve the problem. (F = .10(9N - 1) + 1.00 where F is equal to the fare and N is equal to the number of miles the cab has traveled.) Show and explain how the formula can be used when N = 3 miles.

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STRATEGY 1: cont'd

e.g.,

$$F = .10(9N - 1) + 1.00$$
$$F = .10(9\times3 - 1) + 1.00$$
$$F = .10(27 - 1) + 1.00$$
$$F = .10(26) + 1.00$$
$$F = 2.60 + 1.00$$
$$F = 3.60$$

Repeat this procedure using different mileages or fare rates.

STRATEGY 2: Write the following problem on the chalkboard.

> The post office charges twenty cents for a letter which weighs one ounce. For every ounce over the first, it charges seventeen cents per ounce.

Say, "If you worked for the post office, could you write a formula to help calculate the cost of any letter, when the weight is represented by w?" (cost = 17(w - 1) + 20)

Repeat this procedure for converting a temperature from Celcius to Fahrenheit ($F = \frac{9}{5}C + 32$).

P.O. 7.11.04.01-.02

NAME:______________________CLASS:________DATE:________

Write y = ax or y=ax+b under each equation.

Example: $a = 4s$

$y = ax$

1. $m = 4s + x$

2. $2y + r = g$

3. 2 x radio price = total

4. $m = x + 2r$

5. $n = 3c$

6. $b = 2c + r$

7. cost = 4x shirt price + tax

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Unit 33

7.07.06 CONVERT TO DECIMALS OR FRACTIONS

(2-3 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

7.07.06.01A convert a fraction to a terminating or non-terminating (repeating) decimal fraction; e.g.,

$$\frac{1}{3} = .3333\overline{3} \text{ (repeating)}$$

$$\frac{2}{5} = .4 \text{ (terminating)}$$

7.07.06.02A convert a terminating or non-terminating (repeating) decimal to a fraction. e.g.,

$$.4 = \frac{4}{10} = \frac{2}{5}$$

$$.333\overline{3} = \frac{1}{3}$$

7.07.06

VOCABULARY

TERMS	SYMBOLS
terminating	
non-terminating	
infinite	
repeating bar	$.333\overline{3}$

GETTING STARTED

Write on the chalkboard, $.8\overline{)7.9}$. Say, "What must we do to divide?" (We must change every decimal divisor into a whole number.) Say that we must multiply the divisor and dividend by ten resulting in $8\overline{)79}$ in order to divide.

Ask one student to put a single digit on the chalkboard inside a division symbol, e.g., $\overline{)\ 8}$. Ask another student to pick another single digit, which can be larger or smaller than the number inside, and place it on the outside, e.g., $4\overline{)\ 8}$. Ask the students to find the quotient. Say that a larger number can be divided into a smaller number by placing a decimal point after the digit and adding zeros, e.g., $7\overline{)\ 3}$ can be rewritten as $7\overline{)\ 3.000}$. Ask the students to solve the problems to the thousandths place. Repeat this procedure using other examples.

Write several decimals on the chalkboard and ask the students to read the decimals. Say that by reading each decimal we automatically also read a fraction, e.g., .2 is read as "two tenths" or $\frac{2}{10}$. Practice reading decimal fractions and writing the fraction that is spoken on the chalkboard. Repeat this procedure using serveral other decimals.

Ask the students if they can now reverse the procedure, e.g., write $\frac{5}{10}$ and say and write .5. Place several fractions on the chalkboard and ask the students to write the decimal fractions. Repeat this procedure using other fractions.

7.07.06

7.07.06.01 **convert a fraction to a terminating or non-terminating (repeating) decimal fraction**

STRATEGY 1: Ask the students to write one whole in fractional form, e.g., $\frac{5}{5}$, $\frac{1}{1}$, $\frac{16}{16}$. (N over the same number N is equal to one whole, $\frac{N}{N}$.) Say, "What happens if the number on top is larger than the number on the bottom? (We would have an improper fraction, which is greater than one whole.) What happens if the top number is smaller than the number on the bottom? (We would have a fraction whose value is less than one whole.) All values less than one can be expressed as decimals. When we talk about $\frac{1}{4}$ of a dollar we talk about $.25, and when we talk about $\frac{1}{2}$ of a dollar we talk about $.50. All fractions can be converted to decimals by dividing the numerator by the denominator." Write the following example on the chalkboard.

e.g.,

$$\frac{3}{5} = \begin{array}{r} .6 \\ 5\overline{)3.0} \end{array}$$

Say that we keep adding zeros to the dividend until we can divide completely without any remainder. Write the following fractions on the chalkboard and ask the students to convert to decimals.

e.g.,

$$\frac{1}{4},\ \frac{7}{8},\ \frac{9}{20},\ \frac{4}{5},\ \frac{13}{16},\ \frac{3}{8}$$

Say, "Sometimes when we divide, however, we run into a situation where no matter how many zeros we add, we still cannot divide completely without a remainder. When this happens we have something called a repeating or non-terminating decimal."

STRATEGY 1: cont'd

Write $\frac{1}{3}$ on the chalkboard. Ask the students to convert this to a decimal fraction. Say, "In $\begin{array}{r} .333333 \\ 3\overline{)1.000000} \end{array}$ the 3 repeats forever; this is a case of a repeating decimal. The way we show that a decimal keeps repeating is by putting a bar on top of the part that repeats, e.g., $.33\overline{3}$."

Ask the students to convert the following fractions into decimals, $\frac{5}{9}$, $\frac{5}{6}$, $\frac{9}{11}$, $\frac{5}{12}$.

STRATEGY 2: Write the following on the chalkboard.

e.g.,

Fraction

$$\frac{5}{8} \longrightarrow \begin{array}{r} 0.625 \\ 8\overline{)5.000} \end{array} \longrightarrow \frac{5}{8} = 0.625$$

$$\frac{3}{11} \longrightarrow \begin{array}{r} 0.2727... \\ 11\overline{)3.0000} \end{array} \longrightarrow \frac{3}{11} = 0.27\overline{27}$$

Ask the class to look at the examples and tell why the statement $\frac{5}{8} = 0.625$ is true. Say that the fraction bar means division and that the numerator has been divided by the denominator to get the decimal equivalent. Say that a decimal point and some zeros have been added to the dividend to make the completed division possible.

Write these examples on the chalkboard.

e.g.,

$$\frac{5}{6} \longrightarrow \begin{array}{r} 0.833\overline{3} \\ 6\overline{)5.0000} \end{array} \longrightarrow \frac{5}{6} = .8\overline{3}$$

$$\frac{4}{11} \longrightarrow \begin{array}{r} .3636\overline{36} \\ 11\overline{)4.000000} \end{array} \longrightarrow \frac{4}{11} = .\overline{36}$$

Say, "Look at the example and tell what kind of decimal was found. A repeating decimal is one in which a remainder of zero never occurs. When we have a repeating decimal we place a bar above the digits that repeat."

STRATEGY 2: cont'd Write $\frac{1}{7}$ on the chalkboard and ask a student to convert it to a decimal. Say, "How do we know when a decimal starts repeating itself?" (We know a decimal starts repeating itself when a non-zero remainder appears for the second time in the divisor.)

Repeat this procedure using repeating and non-repeating decimals.

7.07.06.02 **convert a terminating or non-terminating (repeating) decimal to a fraction**

STRATEGY 1: Write .5, .27, .003, five-tenths and twenty-seven hundredths on the chalkboard. Ask the students to read them. Say that when we read a decimal we are automatically reading a fraction, $\frac{5}{10}$, $\frac{27}{100}$, $\frac{3}{1000}$. Write several other decimals on the chalkboard and ask the students to convert the decimals to their equivalent fractions. Say that once we convert a decimal to a fraction, we must reduce the fraction to lowest terms, e.g., $.4 = \frac{4}{10} = \frac{2}{5}$, $3.5 = 3\frac{5}{10} = 3\frac{1}{2}$ ...

Write a repeating decimal on the chalkboard , e.g., $.33\overline{3}$. Say that since $\overline{3}$ means it never ends, we really can't read it as a decimal and must therefore explore a different method to convert this decimal to a fraction. Ask the students what would happen if we multiplied $.33\overline{3}$ by 10. Elicit that the decimal point would move two places to the right forming the decimal $3.3\overline{3}$.

STRATEGY 1: cont'd

Show the students that by using this simple procedure we are now able to solve the problem by doing the following steps.

1) Set x equal to the original decimal. Let $x = .33\overline{3}$.

2) Multiply x by 10, 100, 1000, 10,000 or 100,000 depending upon how many digits there are in the group, e.g., $.\overline{3}$ multiply by 10, $.\overline{27}$ multiply by 100, $.\overline{142857}$ multiply by 1,000,000. Let $10x = 3.33\overline{3}$.

3) Subtract the original decimal from the enlarged decimal, e.g.,

$$\begin{array}{rl} 10\,x = & 3.33\overline{3} \\ -\quad x = & .333 \\ \hline 9\,x = & 3 \end{array}$$

4) Solve for x by dividing both sides by 9.

$$\frac{9x}{9} = \frac{3}{9} \qquad x = \frac{3}{9}$$

5) Reduce the remaining fraction.

$$\frac{3}{9} = \frac{1}{3}$$

Repeat the procedure using $.8\overline{3}$, $.\overline{8}$, $.\overline{27}$ and $.\overline{076923}$.

P.O. 7.07.06.01

NAME:____________________ CLASS:________ DATE:________

Convert the following fractions to terminating or non-terminating decimal fractions.

1. $\frac{2}{5} =$

2. $\frac{9}{20} =$

3. $\frac{3}{25} =$

4. $\frac{5}{4} =$

5. $\frac{5}{6} =$

6. $\frac{7}{9} =$

7. $\frac{5}{6} =$

8. $\frac{5}{12} =$

P.O. 7.07.06.02

NAME:____________________ CLASS:________ DATE:________

Convert the following decimal fractions to fractions.

1. $.25 =$

2. $.075 =$

3. $.250 =$

4. $.016 =$

5. $0.2727\overline{27} =$

6. $0.55\overline{5} =$

7. $.8181\overline{81} =$

8. $.416\overline{6} =$

7.06.04 MEANING OF PERCENT

(4 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

7.06.04.01A convert a fraction whose denominator is a factor of 100 to a decimal and a percent; e.g.,

$$\frac{2}{5} = \frac{40}{100} = 40\%$$

7.06.04.02A convert a mixed number to a decimal and a percent; e.g.,

$$1\frac{1}{2} = \frac{3}{2} = \overset{1.50}{2\overline{)3.00}} = 150\%$$

7.06.04.03A convert a whole number percent to a decimal fraction. e.g.,

$$65\% = .65$$

7.06.04

VOCABULARY

TERMS	SYMBOLS
percent	%

GETTING STARTED

Write 23.462 on the chalkboard. Say, "How do we read this number? (twenty-three and four hundred sixty-two thousandths) Which number is in the tenths place? (4) hundredths place? (6) thousandths place?" (2)

Repeat this procedure using 9.5, 63.48, 189.0163.

Draw a place value chart on the chalkboard through ten thousandths. Ask a student to write .009 in the chart in its correct place value.

Repeat this procedure using 2.136, 8.59, 0.2001.

Write $\frac{3}{5}$ on the chalkboard. Ask a student to convert this fraction to a fraction whose denominator is 10, e.g., $\frac{3}{5} \times \frac{2}{2} = \frac{6}{10}$. Repeat this procedure for $\frac{1}{2}$, $\frac{2}{5}$, $\frac{4}{5}$. Ask the students to convert each fraction (as tenths) to a decimal, e.g., $\frac{6}{10} = 0.6$

7.06.04

7.06.04.01 convert a fraction whose denominator is a factor of 100 to a decimal and a percent

STRATEGY 1: Write the word percent on the chalkboard. Ask the students if they know what the word percent means. (Percent means per one hundred or for every hundred.) Write 37% on the chalkboard and say, "This means 37 per 100." Repeat this procedure using other examples, e.g., 14% of the people play tennis. Say, "How can we write this a shorter way?" (14 out of 100 or $\frac{14}{100}$)

Write several fractions with denominators of 100 on the chalkboard. Ask the students to name each as a decimal and describe why writing a fraction with a denominator of 100 as a decimal is so easy.

e.g.,

$$\frac{8}{100} = .08 \qquad \frac{25}{100} = .25 \qquad \frac{16}{100} = .16$$

Write $\frac{18}{40}$ on the chalkboard. Say, "How can we write this fraction as a decimal?" Write the following two methods on the chalkboard.

e.g.,

$$\text{A)}\ \begin{aligned} \frac{18}{40} &= \frac{9}{20} = \frac{45}{100} \\ &= .45 \\ &= 45\% \end{aligned} \qquad\qquad \text{B)}\ \begin{aligned} \frac{18}{40} &= 40\overset{.45}{\overline{)18.00}} \\ &= .45 \\ &= 45\% \end{aligned}$$

Say that when the denominators divide into 100 evenly, it is easier to convert to a decimal. Say, "In order to change a decimal to a percent, we multiply by one hundred and use a percent sign (%), e.g., .45 x 100 = 45%." Ask the students to convert each of the above decimals to a percent. Continue this procedure using similar examples.

STRATEGY 2: Ask the class to list all the factors of 100. (1, 2, 4, 5, 10, 20, 25, 50, 100) Write several

STRATEGY 2: cont'd

fractions on the chalkboard using these factors as denominators.

e.g.,

$$\frac{1}{2}, \frac{3}{4}, \frac{4}{5}, \frac{7}{10}, \frac{19}{20}, \frac{14}{25}, \frac{37}{50} \text{ and } \frac{19}{100}.$$

Ask the class to convert these fractions into equivalent fractions with the denominator of 100. ($\frac{1}{2} = \frac{50}{100}$, $\frac{3}{4} = \frac{75}{100}$, $\frac{4}{5} = \frac{80}{100}$, $\frac{7}{10} = \frac{70}{100}$, $\frac{19}{20} = \frac{95}{100}$, $\frac{14}{25} = \frac{56}{100}$, $\frac{37}{50} = \frac{74}{100}$ and $\frac{19}{100} = \frac{19}{100}$) Say, "When we have a fraction whose denominator is a factor of 100, we can simply rename the fraction as a decimal. $\frac{50}{100}$ is 50 hundredths which can be written as .50. Rename the fractions above as decimals." Ask half of the class to create their own fractions with the factors of 100 as denominators, and ask the other half of the class to rename them as percentages.

Say, "A percent is a ratio of one number to a hundred, e.g., $\frac{37}{100}$ is 37% of one hundred. In order to convert a fraction to a percent, we must convert the fraction to a hundredths place decimal and multiply the decimal by one hundred. We add a percent (%) sign to show it is percent."

e.g.,

$$\frac{1}{2} = \frac{50}{100} = .50 = (.50 \times 100) = 50\%$$

Repeat this procedure using the above fractions.

7.06.04.02 **convert a mixed number to a decimal and a percent**

STRATEGY 1: Ask a student if he/she can tell what a decimal point does. (A decimal point separates whole numbers from fractional parts.) Say that 3.45 means 3 wholes and 45 hundredths. Say that the 3 in 3.45 is the same as the 3 in $3\frac{45}{100}$. Say, "In order to change a mixed number to a decimal, we

STRATEGY 1: cont'd

take the whole number part of the mixed number, place it to the left of the decimal point and convert the fraction to a decimal." Show the following examples.

$$3\frac{2}{5} = 3.40 \qquad 17\frac{1}{4} = 17.25$$

$$\frac{2}{5} = .40 \qquad \frac{1}{4} = .25$$

Write several mixed numbers on the chalkboard and ask the students to convert them to decimals.

Say, "Who remembers how to change a decimal to a percent?" (We multiply the decimal by 100 and add on a percent sign.)

e.g.,

$$.32 = .32 \times 100 = 32\%$$
$$6.37 = 6.37 \times 100 = 637\%$$
$$.25 = .25 \times 100 = 25\%$$

Ask the class if they can now convert a mixed number to a percent by going through these steps.

e.g.,

$$3\frac{1}{4} = 3 + \frac{1}{4} = 3.25 \times 100 = 325\%$$

Write several other mixed numbers on the chalkboard and ask the class to convert them to percentages.

STRATEGY 2: Ask the class if they remember how to change a fraction to a percent. (We divide the denominator into the numerator and multiply the result by 100.)

e.g.,

$$\frac{4}{5} = 5\overset{.80}{\overline{)4.00}} \times 100 = 80\%$$

Say, "If we convert a fraction to a percent using this rule, how would we convert a mixed number such as $2\frac{3}{4}$ to a percent?" (We could convert a mixed number to an improper fraction and then use the rule above.) Ask a student to come to the chalkboard and change $2\frac{3}{4}$ to a percent.

e.g.,

$$2\frac{3}{4} = \frac{11}{4} = 4\overset{2.75}{\overline{)11.00}} \times 100 = 275\%$$

Ask several students to do other problems.

7.06.04

7.06.04.03 convert a whole number percent to a decimal fraction

STRATEGY 1: Say, "The other day I was given a multiplication problem to do. The problem was 20% x .14. How can I do this problem?" (The problem cannot be done in this form because we cannot multiply percentages by decimals. In order to do this problem we must first change 20% into an equivalent decimal.) Say that every percent can be renamed in other ways.

e.g.,

$$20\% = 20 \times .01$$
$$20\% = 20 \times \frac{1}{100}$$
$$20\% = \frac{20}{100}$$
$$20\% = .20$$

$$\begin{array}{r} .20 \\ \times.14 \\ \hline 80 \\ 200 \\ \hline .280 \end{array}$$

Say that 1% means one out of a hundred.

Write another example on the chalkboard.

e.g.,

$$260\% = 260 \times .01$$
$$260\% = 260 \times \frac{1}{100}$$
$$260\% = \frac{260}{100}$$
$$260\% = 2.60$$

Say, "This method always works, but it is easier to remember that whenever you see a number with a percent sign next to it, you are seeing a decimal that has already been multiplied by 100, e.g., 7% can be renamed as .07 x 100. To change a percent to a decimal, we move the decimal point two places to the left."

e.g.,

$$85\% = 85.\% = .\underbrace{8\;5}.$$
$$115\% = 115.\% = 1.\underbrace{1\;5}.$$

Write several percents on the chalkboard and ask the students to come to the chalkboard and change them to decimals.

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7.06.04

STRATEGY 2: Ask what 82% means. (82% means 82 out of 100.) Ask a student if he/she could think of another way to write 82 out of 100. (82 out of 100 can be written as a ratio, $\frac{82}{100}$, or as .82, which means 82 hundredths.)

Write the following chart on the chalkboard.

e.g.,

Percent	Decimal
36%	
45%	
327%	

Ask the students to convert each percent to a decimal.

P.C. 7.06.04.01

NAME: ______________________ **CLASS:** __________ **DATE:** __________

Write each fraction as a percent.

1 $\frac{1}{2} =$

2 $\frac{1}{5} =$

3 $\frac{7}{10} =$

4 $\frac{3}{20} =$

5 $\frac{9}{25} =$

6 $\frac{17}{25} =$

7 $\frac{4}{5} =$

8 $\frac{1}{50} =$

9 $\frac{7}{50} =$

10 $\frac{50}{50} =$

11 $\frac{1}{50} =$

12 $\frac{27}{20} =$

13 $\frac{19}{20} =$

14 $\frac{13}{10} =$

15 $\frac{1}{10} =$

16 $\frac{19}{25} =$

32

P.O. 7.06.04.02

NAME:____________________ CLASS:__________ DATE:________

Convert these mixed numbers to percents.

1 $8\frac{1}{4} =$	2 $4\frac{2}{5} =$	3 $6\frac{3}{4} =$	4 $9\frac{8}{10} =$
5 $4\frac{7}{8} =$	6 $10\frac{1}{2} =$	7 $11\frac{2}{3} =$	8 $18\frac{4}{5} =$
9 $8\frac{3}{4} =$	10 $2\frac{3}{5} =$	11 $14\frac{2}{12} =$	12 $3\frac{3}{10} =$
13 14	14 $7\frac{7}{11} =$	15 $8\frac{3}{6} =$	16 $14\frac{2}{7} =$

P.O. 7.06.04.03

NAME:________________ CLASS:________ DATE:________

Write each percent as a decimal fraction.

1 13% = ______

2 17% = ______

3 29% = ______

4 21% = ______

5 50% = ______

6 49% = ______

7 87% = ______

8 53% = ______

9 87% = ______

10 89% = ______

11 99% = ______

12 114% = ______

13 151% = ______

14 117% = ______

15 225% = ______

16 379% = ______

7.06.05 VERBAL PROBLEMS INVOLVING PERCENT

(4 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

7.06.05.01A solve a verbal problem to find the amount of discount;

e.g.,

Find the discount price of a cap which costs $1.50, if the discount is 10%.

$$\begin{array}{r} \$1.50 \\ \underline{\times\ .10} \\ .1500 \end{array}$$

The discount is $.15.

7.06.05.02A solve a verbal problem to find the amount of sales tax;

e.g.,

Find the total cost of an item which costs $7.95 if the sales tax is 6%.

$$\begin{array}{r} \$7.95 \\ \underline{\times\ .06} \\ .4770 \end{array}$$

Sales tax = $.48

Total cost = $7.95 + .48 = $8.43

7.06.05.03A solve a verbal problem to find simple interest (Interest = Principal x Rate x Time).

e.g.,

A man borrowed $500 for 1 year
at 5% interest per year.
How much interest must he pay
on the loan?

$$\begin{array}{rl} \$500 & \\ \underline{\times\ .05} & \\ \$25.00 & \\ \underline{\times\quad\ 1} & \underline{\text{(year)}} \\ \$25.00 & \text{interest} \end{array}$$

7.06.05

VOCABULARY

TERMS

discount
list price
sale price
net amount
sales tax
interest
principal
simple interest
rate of interest

GETTING STARTED

Write the following example on the chalkboard.

e.g.,

$$\begin{array}{r} \$72.50 \\ \underline{\times\ .06} \end{array}$$

Ask a student to come to the chalkboard to calculate the answer. ($4.35) Say that we must have four decimal places in the answer because there are four decimal places in the problem.

Repeat this procedure using other examples.

e.g.,

$$\begin{array}{r} 64.50 \\ \underline{\times\ .50} \end{array} \qquad \begin{array}{r} 123.75 \\ \underline{\times\ .03} \end{array} \qquad \begin{array}{r} 49.95 \\ \underline{\times\ .035} \end{array} \qquad \begin{array}{r} 23.5 \\ \underline{\times\ .08} \end{array}$$

Write a percent on the chalkboard, e.g., 60%. Ask the students if anyone can change this into an equivalent decimal.

e.g.,

$$60\% = .60 \text{ or } \frac{60}{100}$$

Repeat this procedure using other examples.

e.g.,

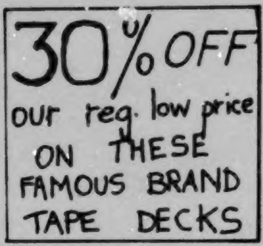
75% = ☐
3% = ☐
15% = ☐
43% = ☐

7.06.05

7.06.05.01 solve a verbal problem to find the amount of discount

STRATEGY 1: Prepare and display newspaper advertisements of discounts or sales that are written as percentages.

e.g.,



Say, "A discount is a savings off the regular price. Let's pretend you wanted to buy a tape deck that regularly sold for $100.00. The discount is 30%. How much money will you save?" Write 30% on the chalkboard. Ask a student to write 30% as a decimal. (30% = .30)

Say, "To find the amount that we save, we must multiply the amount of the item by the discount."

e.g.,

$$\begin{array}{r}\$100\\ \underline{x.30}\\ \$30.00\end{array}$$

Repeat this procedure using various discounts and prices.

STRATEGY 2: Say, "I went on a shopping trip. I saw a beautiful jacket that cost $100.00. Above the rack there was a sign that said 25% off. How much money would I save?" Write 25% on the chalkboard. Ask a student to give you the equivalent of 25% in decimal form. (25% = .25) Explain that to find the amount that is saved we must multiply .25 by the original price. Write $\begin{array}{r}100\\ \underline{x.25}\end{array}$ on the chalkboard. Ask a student to solve the problem.

Repeat this procedure using various discounts and prices.

7.06.05

7.06.05.02 solve a verbal problem to find the amount of sales tax

STRATEGY 1: Say, "If you went to the store and wanted to purchase a clock for $35.00 and you had exactly $35.00 in your wallet, could you purchase the clock?" (No, since $35.00 is the price of the clock without the sales tax.)

Say, "If the sales tax is 8%, we must multiply that by the cost of the clock to find out the sales tax." Ask a student to convert 8% to a decimal and find the tax. (8% = .08)

e.g.,

$$\begin{array}{r} 35 \\ \underline{\times .08} \\ \$2.80 \end{array}$$

Say, "$2.80 is the sales tax that must be added to the price of the clock."

e.g.,

$$\begin{array}{rl} \$35.00 & \text{price} \\ \underline{+\ 2.80} & \text{tax} \\ \$37.80 & \end{array}$$

Repeat this procedure using various prices and taxes.

STRATEGY 2: Say, "If you went shopping for a pair of sneakers and you had $30.00 to spend, could you buy sneakers that were $29.00 with the tax at 5%? (29.00 x .05 = $1.45 and $29.00 + 1.45 = 30.45) $30.00 is not enough to buy the sneakers."

Repeat this procedure using various prices and percentages.

7.06.05

7.06.05.03 **solve a verbal problem to find simple interest (Interest = Principal x Rate x Time)**

STRATEGY 1: Write the following formula on the chalkboard.

$$\text{Interest} = \text{Principal} \times \text{Rate} \times \text{Time}$$
$$I = PRT$$

Say, "Interest is money paid for the use of money. The money borrowed on which there is interest is called the principal. The rate of interest is a percent paid per year unless specified otherwise. To find the interest, multiply the principal by the rate of interest. Then multiply this by the time expressed in years."

e.g.,

$$I = P \times R \times T$$
$$I = \$900 \times 4\% \times 1 \text{ year}$$
$$I = \$900 \times .04 \times 1$$
$$I = \$36$$

Repeat this procedure using $600 for 1 year at 5%. Say, "What if the time was not a full year but only 6 months? Six months is one half of a year."

e.g.,

$$I = 600 \times .05 \times \frac{1}{2}$$
$$I = 30 \times \frac{1}{2}$$
$$I = \frac{30}{2} = 15$$
$$I = \$15$$

Repeat using various principals, rates and times.

STRATEGY 2: Say, "If we borrow money we are charged an amount for the use of the money. This amount is called interest. The amount of interest is based on how much you are borrowing. This amount you are borrowing is called the principal. Interest is also based on for how long you want to borrow the money. (time) Each bank or other organization has

STRATEGY 2: cont'd

a different rate that they charge. This rate is expressed as a percent."

e.g.,

Interest = Principal x Rate x Time

I = PxRxT

Say, "We are going to borrow $1000 at 6% interest for one year. We must convert the 6% to a decimal."

e.g.,

I = $1000 x .06 x 1

I = $60.00

Repeat this procedure using other interest problems.

P.O. 7.06.05.01

NAME:________________________________ CLASS:__________ DATE:________

How much do you save?

Sale - 10% off

Price	Amount Saved
Example	
$12.50	$1.25
$63.00	
$153.00	
$9.50	
$23.50	

Discount - 25% off

Price	Amount Saved
$16.00	
$130.00	
$250.00	
$33.00	
$45.00	

Save - 30%

Price	Amount Saved
$42.50	
$17.50	
$29.00	
$125.00	
$2.00	

Super-Sale at Bloom
30% OFF!
$42.50
$45
$125
29

P.O. 7.06.05.02

NAME:______________________________ CLASS:__________ DATE:________

Find the total cost for each problem below.

Example

1. Mr. Wilson bought a camera for $56.00. The sales tax is 7%. What is the total price of the camera?

$$\begin{array}{rl} 56.00 & \\ \underline{.07} & \\ 3.9200 & \text{tax} \end{array} \qquad \begin{array}{rl} 56.00 & \\ \underline{3.92} & \text{tax} \\ \$59.92 & \text{total cost} \end{array}$$

2. A jacket costs $39.00 the sales tax is 6%. What is the total cost of the jacket?

3. Miss Jones wanted a picture that costs $65.50. The sales tax is 8%. Miss Jones has $70.00. Does she have enough money to purchase the picture?

Item Price	Tax	Total Price
A radio costs $101.50	8%	
A coat costs $72.00	6%	
A sweater costs $31.20	5%	
A record costs $6.00	7%	
A vase costs $15.50	6%	

P.O. 7.06.05.03

NAME:______________________________CLASS:________DATE:________

Find the interest.

Principal	Rate	Time	Interest
$200	6%	1 yr.	200 x .06 x 1 = $12.00
$1,250	6%	1 yr.	
$175	7%	6 mos.	
$3,000	9%	2 yrs.	
$5,400	8%	1 yr.	
$10,000	9%	3 yrs.	

7.12.03 MULTIPLE GRAPHS

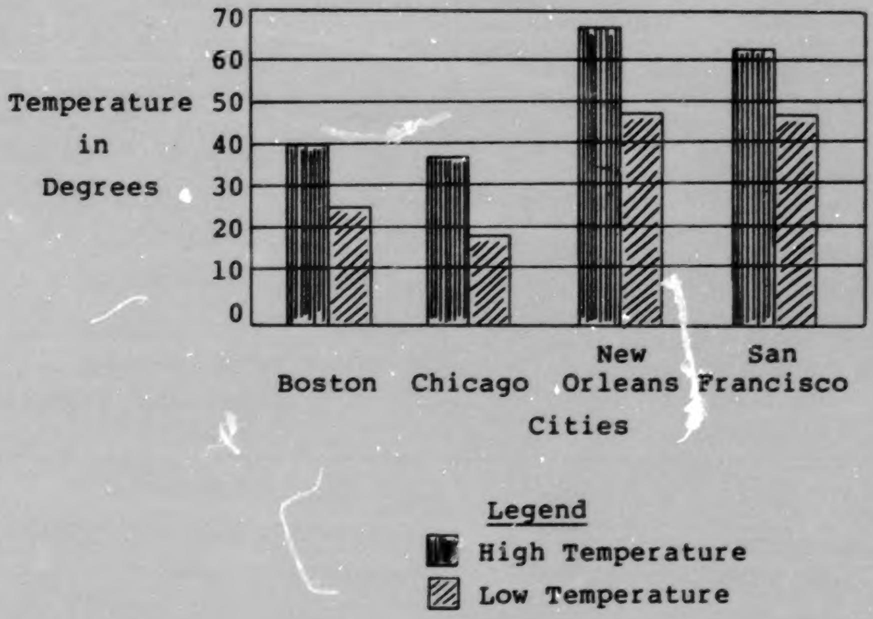
(2-3 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

7.12.03.01B construct, read and interpret a multiple bar graph (involving a two bar comparison); e.g.,

High and Low Normal Temperatures for January in Several United States Cities

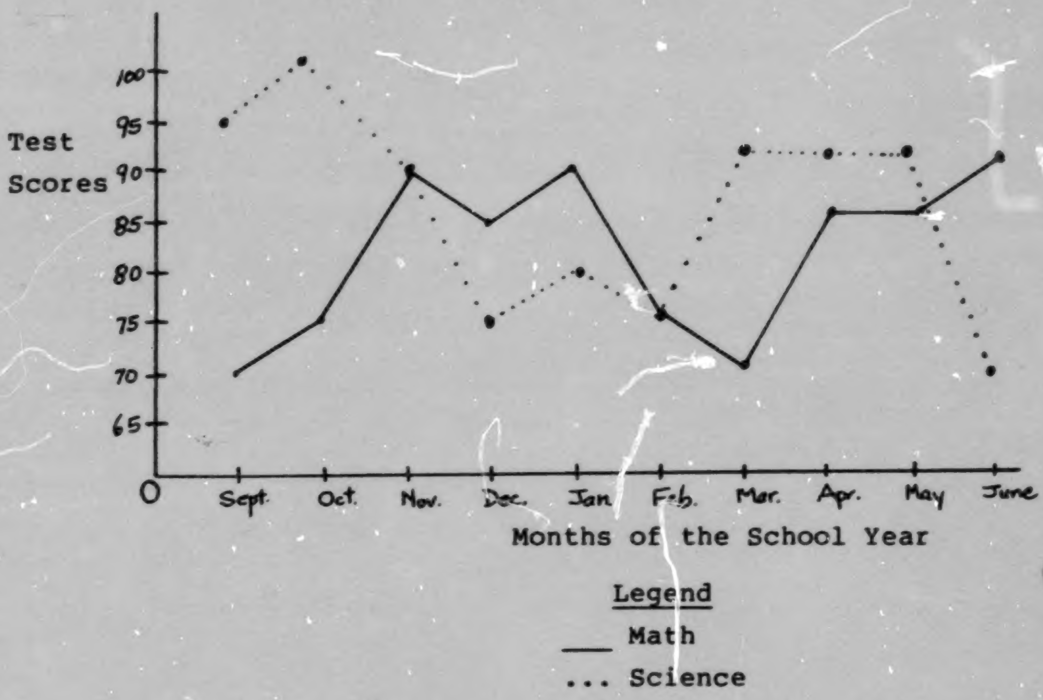


What was the high temperature in Boston in January? What was the low temperature?

7.12.03.02B read and interpret a multiple line graph (involving a two line comparison);

e.g.,

Susan's Monthly Test Scores in Mathematics and Science

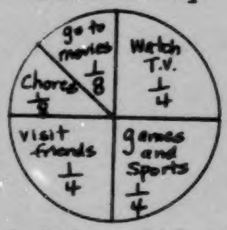


What was the difference in Susan's mathematics scores from March to April?

7.12.03.03B construct, read and interpret a circle graph (involving fractional or percent comparisons).

e.g.,

Bill's Day



On which activities did Bill spend the same amount of time?

7.12.03

VOCABULARY

TERMS

multiple bar graph
multiple line graph

GETTING STARTED

Prepare graphs from newspapers, Science, Social Studies and Math books. Distribute a graph to each student. Ask the following questions.

e.g.,
1) What is the title of the graph?
2) What is the title of the horizontal axis?
3) What is the title of the vertical axis?
4) What is the interval (scale) between numbers?
5) Does the scale on line graphs begin at zero?
6) What are the measurements used in a circle graph? (fractions and percents)
7) Are all graphs accurate? (Sometimes numbers are rounded off.)
8) What advantage do graphs have over other tables of data?

Say that you are going to make a survey of the students' favorite TV programs. Separate data for boys and girls.

e.g.,

Favorite TV Program

Name of Show	Boys	Girls

Collect the data and mark it on the chart. Ask the students to answer questions from the table.

e.g.,
1) Which show is the most popular? ...boys' favorite? ...girls' favorite?
2) Which show is the least favorite? ...boys' least favorite? ...girls' least favorite?

Ask a student to name another way to show this information. Elicit that another way is with a graph (bar, line or circle).

7.12.03

7.12.03.01 **construct, read and interpret a multiple bar graph (involving a two bar comparison)**

STRATEGY 1: Write the following table on the chalkboard.

e.g.,

TRAFFIC ACCIDENTS

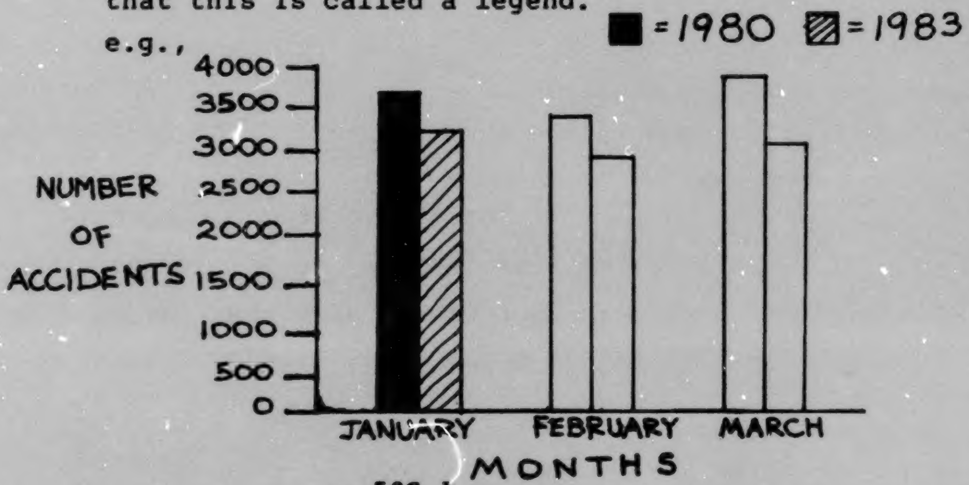
Months	Years	
	1980	1983
January	3,650	3,260
February	3,440	2,940
March	3,810	3,170

Ask the students to construct a multiple bar graph using this table, a ruler, graph paper and a pencil. Say that you need to round off the numbers to the nearest 250 to use them in a graph.

Say, "The scale must start at zero. What intervals should we use?" (five hundred) Write them on the chalkboard. Proceed until the scale reaches 4000 on the vertical axis. Say, "What shall we label this axis?" (Number of Accidents) Ask a student what information will go on the horizontal axis. (Months, e.g., Jan., Feb., March) Label the axis.

Say that the bars must be the same width as the space between. Ask the students to use different designations, e.g., solid, lines, dots for the bars. Ask the students to explain on their graphs the meaning of solid ■ = 1980, ▨ = 1983. Say that this is called a legend.

e.g.,



STRATEGY 1: cont'd

Stop at a point in the above diagram and ask a student to complete the graph. After the graph is complete ask the students to make up five questions that can be answered by this graph, e.g.,

a) What month or months had the most traffic accidents in 1980?

b) How do these numbers compare with the same months in 1983?

Ask several students to present their questions for class discussion.

STRATEGY 2:

Say, "Why would we use a multiple bar graph?" (To show two sets of data simultaneously.) Ask the students to suggest topics for a multiple bar graph, e.g., favorite sports (boys, girls). Ask the students to construct a table of data on the chalkboard. Construct a multiple bar graph on the chalkboard while the class constructs theirs in their notebooks. Say, "What should we call the graph? What other labels are necessary? What interval (scale) should we use?" Start the construction and then ask a student to complete the graph at the chalkboard. Ask the students to make up questions about the information on the graph.

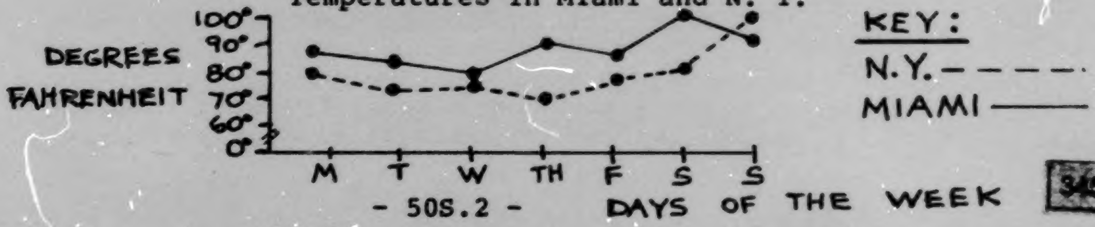
7.12.03.02 **read and interpret a multiple line graph (involving a two line comparison**

STRATEGY 1:

Ask the students what would happen if two line graphs were combined on one graph. (We could get confused unless we were able to differentiate between the two sets of data.) Place the following graph on the chalkboard.

e.g.,

Temperatures in Miami and N. Y.



7.12.03

STRATEGY 1: Ask the following questions.
cont'd

1) What is the title of the graph?
2) How can we differentiate between the temperatures in Miami and N. Y.? Where do you get this information?
3) On what day was it warmest in N. Y.?
4) On the average, how many degrees hotter is it in Miami every day?
5) On which day was it the hottest in New York?
6) Does the scale start at zero?

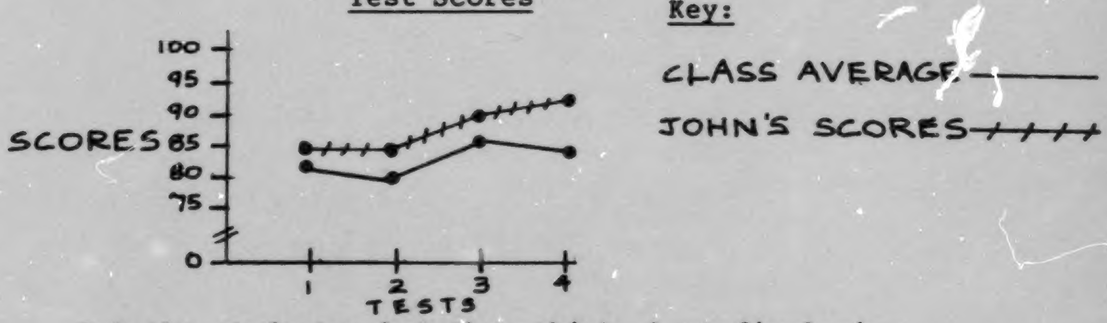
STRATEGY 2: Place the following data on the chalkboard.

On the last four tests the class averages were: test 1 (82), test 2 (80), test 3 (86) and test 4 (85). John's test marks were 85, 85, 90, 92.

Ask the students if they could represent this data in other ways. (Graphing is a viable alternative.) Ask the students if they think they can graph all of this information on one graph. (In order to combine the information on one graph, we will have to differentiate the class scores from John's scores.) Ask the students what are the different ways to differentiate lines in a line graph. (broken line, solid line, blue line, red line, dotted line...) Draw the following graph on the chalkboard.

e.g.,

Test Scores



Ask the students what they think about displaying the data in this fashion. Ask the following questions.

1) Why would we display the data this way instead of 2 single line graphs?

345

7.12.03

STRATEGY 2: cont'd

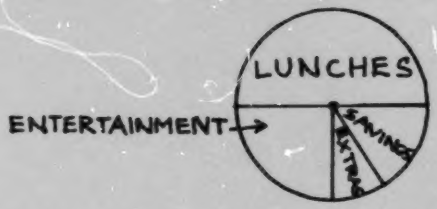
2) Is it easier to compare data in a multiple line graph? Why or why not?
3) What can you say about John as a student?
4) Does the scale on a line graph have to start at zero?

Ask the students to make up their own questions about this graph.

7.12.03.03 **construct, read and interpret a circle graph (involving fractional or percent comparisons)**

STRATEGY 1: Place the following circle graph on the chalkboard. e.g.,

Maria's Allowance



Say that a circle graph is one way to show how the whole is divided into fractional parts or percents. Ask the following questions.

1) What is Maria's largest expense?
2) Is more spent on entertainment or extras?
3) On which items are the same amount spent?
4) If Maria's allowance is eight dollars, how much money is spent on each item?

Ask the students to construct a circle graph using another topic. Elicit the questions that should be on the survey. Use the class for the survey. Construct a table of data. Ask the students to draw a circle graph showing the results. Ask them to round off the percents to 50%, 25%, 10%.) Ask several students to read and interpret the information given on the graph.

P.O. 7.12.03.01

NAME:______________________________CLASS:__________DATE:______

Construct a multiple bar graph from the table. Make up questions from the graph.

First-quarter nuptials down across the U.S.A.

(January-March)

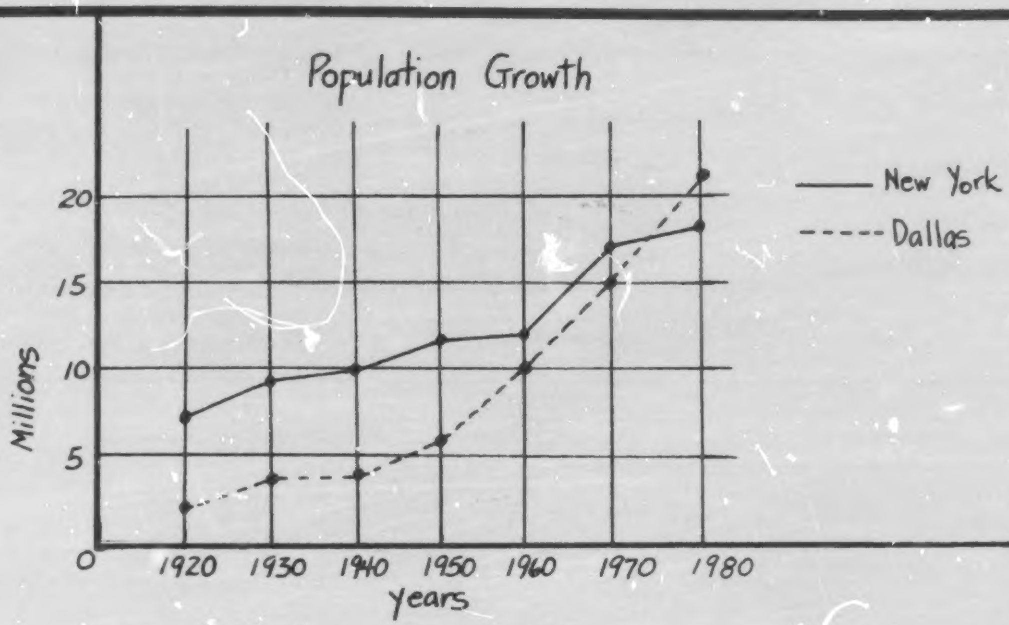
STATE	1981	1983
Ala.	10,895	10,388
Alaska	1,219	1,706
Ariz.	7,302	7,052
Ark.	4,661	5,927
Calif.	39,742	42,972
Colo.	7,058	6,937
Conn.	3,523	3,200
Del.	750	886
Dec.	940	887
Fla.	25,642	27,073

348

P.O. 7.12.03.02

NAME:____CLASS:_________DATE:_______

Answer the questions under the graph.



1. The population growth of which two states are shown?
2. What was the approximate population of Dallas in 1960?
3. What was the approximate population of N.Y. in 1930?
4. How much greater was the population of N.Y. than Dallas in 1960?
5. What are the chances that the population will keep increasing in N.Y.? in Dallas ?
6. Which city is increasing in population more quickly?

P.O. 7.12.03.03

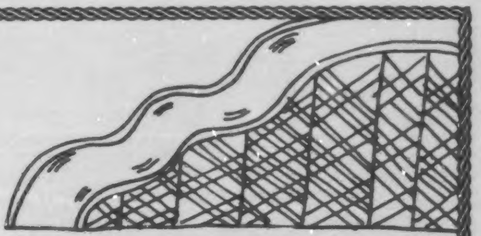
NAME:____CLASS:__________DATE:________

Construct a circle graph from the table of information below.

Most Popular Rides In Amusement Parks!

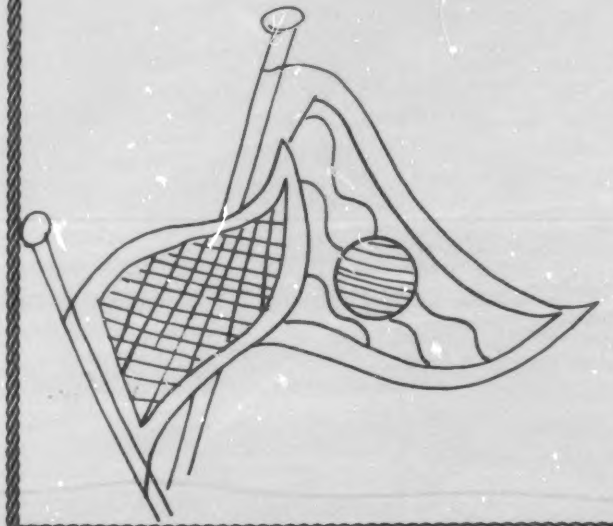
Astroland
Coney Island

Great Adventure
New Jersey



Rides	Number of People	Fractions and Percents
Roller Coaster	40	$\frac{40}{100} = \boxed{}$ %
Free Fall	30	
Giant Ferris Wheel	10	
Parachute Jump	20	

100 people surveyed



7.13.03 GUESS AND CHECK

(1-2 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

*7.13.03.01A make and refine a guess about his/her environment;

e.g.,

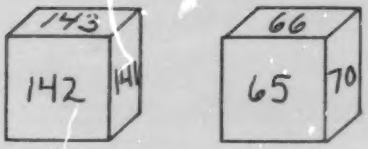
How many times does your pulse beat in one hour?

7.13.03.02A check guesses against problem conditions.

e.g.,

The cubes below are numbered consecutively.

Find the largest even product obtained by rolling both cubes.



GETTING STARTED

Prepare and display various weights. Ask a student to determine by holding three of the weights, which one is the heaviest, the lightest, the one in the middle. Say, "Why do you feel this weight is the middle weight?" (It is lighter than one but heavier than the other.) Repeat this with different weights.

Prepare a container filled with beans which you have counted. Ask each student to guess the number of beans in the container. Write the students' guesses on the chalkboard. Open the jar and begin counting the beans for several minutes. Ask the students if they would like to change their guesses. Record the second guess next to the first guess. Compare the guesses with the actual count and say whose guess was closest.

7.13.03

7.13.03.01 **make and refine a guess about his/her environment**

STRATEGY 1: Ask three students to stand in front of the class. Be sure they are three different heights. Say, "John is the tallest, Paul is the shortest, David is between John and Paul. Let's see if we can guess how tall David is. Besides measuring David, how could we guess how tall he is?" (Measure John and measure Paul. Since David is between them, we will know between what two measurements he is.)

Measure John and write his height on the chalkboard, e.g., John 5 ft. 2 in. Repeat for Paul. Say, "We now know that David is between 5 ft. 2 in and 4 ft. 9 in. Is David closer to Paul or John? How much taller is David than Paul? We know that Paul is 4 ft. 9 in., what would we guess David's height to be?" Now measure David. Determine how accurate the guess was.

Repeat this procedure using various examples.

STRATEGY 2: Fill three containers with different temperatures of water. Ask a student to decide which is the warmest, coldest and the one in between. Ask the student why he selected the middle one. (The middle was cooler than one and warmer than the other. Ask a student to measure the temperature of the warmest and the coldest.) Record the two temperatures on the chalkboard.

e.g.,

warmest 80° coolest 50°

Ask several students to feel all three and ask if the middle one is closest to 80° or 50°. Ask students to guess how close. Measure the temperature of the middle container and record the rsults.

Say that narrowing down or refining a guess makes the final guess very close.

7.13.03

7.13.03.02 check guesses against problem conditions

STRATEGY 1: Say, "Susan bought skirts for $16 each and blouses for $10 each. She spent 98 dollars. How many skirts and blouses did she buy?"

Ask the students to work out various combinations to see which one makes a total of $98.00. Write a chart on the chalkboard and fill in amounts as students give information.

e.g.,

No. of blouses	1	2	3	4	5	6	7	8	9
Cost of blouse	10	20	30	40	50	60	70	80	90
Amt. remaining	88	78	68	58	48	38	28	18	8
No. of skirts	-	-	-	-	3	-	-	-	-

After working out the various combinations, elicit that Susan bought five blouses for $50 and three skirts for $48.

Repeat this procedure using various examples.

STRATEGY 2: Write the following on the chalkboard.

> Mark's mother gave him 25¢ each day his room was cleaned. Mark had to pay his mom 15¢ each day it wasn't cleaned. At the end of a certain amount of time, Mark's mom paid him 20¢. How many days did he clean his room?

Ask the students to make a chart and fill in the various amounts.

e.g.,

Number	1	2	3	4	5	6
Amount for clean room	25¢	50¢	75¢	1.00	1.25	1.50
Amount for messy room	15¢	30¢	45¢	60¢	75¢	90¢
Difference	10¢	20¢	30¢	40¢	50¢	60¢

Continue the chart for ten days. Ask students to find what combinations of clean and messy rooms would give Mark 20¢ profit.

Repeat this procedure using various examples.

P.O. 7.13.03.01 - .02

NAME: ______________________________ CLASS: __________ DATE: ________

Solve each problem.

Example -

Name 2 whole numbers, which when added equal 118 and when the smaller is subtracted from the larger the difference is 36.

100 + 18 = 118   90 + 28 = 118   82 + 36 = 118
100 - 18 = 82   90 - 28 = 62   82 - 36 = 46

77 + 41 = 118
77 - 41 = 36

77 and 41

1. Two whole numbers when added are 45. When multiplied they equal 486. What are the numbers?

_______ and _______

2. Sam has a collection of nickels and quarters. There are 2 more nickels than quarters. How many nickels and quarters are there if Sam has $3.40?

_______ quarters and _______ nickels

3. Mike has blue and red marbles. If you multiply the number of red and blue you get 195, if you add the numbers you get 28. How many red and blue marbles does he have?

_______ red and _______ blue

7.10.04 PERIMETER, AREA AND VOLUME

(5-6 lessons)

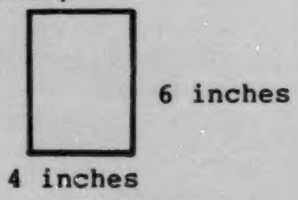
PERFORMANCE OBJECTIVES

The student will be able to...

7.10.04.01A use the formulas for finding the perimeter of a rectangle, triangle, square; e.g.,

Square : $P = 4s$
Rectangle : $P = (2 \times L) + (2 \times W)$

7.10.04.02A find the area of polygons by using the appropriate formulas; e.g.,



Area = 6 x 4 = 24 square inches

7.10.04.03A compute the volume of a rectangular prism and a cube in standard units of measure using the formulas, $V = L \times W \times H$ and $V = B \times H$; e.g.,

$V = 4 \text{ ft.} \times 5 \text{ ft.} \times 3 \text{ ft.}$
$V = 60$ cubic feet

7.10.04.04A solve verbal problems related to skills learned in this module.

e.g.,

If 1 gallon of paint covers 400 square feet of wall and 1 quart covers 100 square feet, how much paint should Lorraine buy to paint a wall which is 25 feet by 20 feet?

$$\begin{array}{r} 20 \\ \underline{\times 25} \\ 500 \end{array} \text{ square feet}$$

1 gallon and 1 quart of paint (400 + 100)

7.10.04

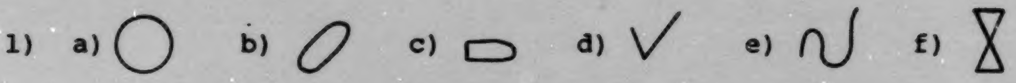
VOCABULARY

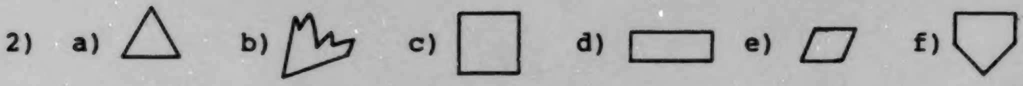
TERMS

polygon
parallelogram
trapezoid
volume
plane figure
rectangular prism
cube

GETTING STARTED

Draw the following on the chalkboard.

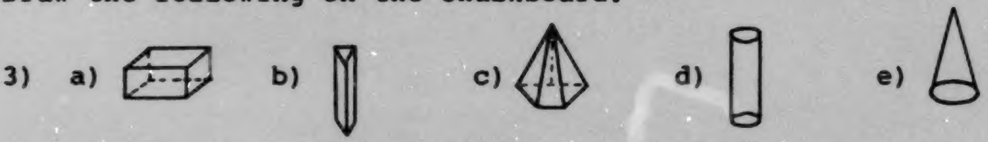




Ask the students to determine what all the above figures have in common. (All are open or closed and all are plane figures.)

Ask the students to determine how the figures in 1 and 2 are different. (The figures in 2 are all simple closed curves formed by line segments, while those in 1 are not.) Say that the figures in 2 are called polygons. Ask the students to name which of these polygons is a triangle, which is a square, which is a rectangle and which are parallelograms.

Draw the following on the chalkboard.



Ask the students how the figures in 3 differ from the previous figures in 1 and 2. (These are space or solid figures.)

GETTING STARTED cont'd

Ask the students which of these figures are prisms. (a and b) Ask which are rectangular prisms. (a) Say that a rectangular prism with all edges equal is called a cube.

Ask students to come to the chalkboard to draw examples of plane figures. Ask the students which, if any, of the figures are polygons. Say that polygons are simple closed curves formed by line segments. If none of the drawn figures are polygons, ask students to draw some on the chalkboard.

Ask the students which, if any, of the drawn figures are triangles, squares, rectangles or other parallelograms. Ask the students to draw any of these figures on the chalkboard that are not already there.

Ask several students to come to the chalkboard to draw space (solid) figures. Ask the students if any of the drawn figures are prisms. If not, ask the students to draw some. Ask the students to identify any rectangular prisms or cubes (rectangular prism with equal edges). Ask the students to draw any of these figures that are not already on the chalkboard.

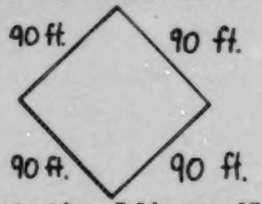
7.10.04

7.10.04.01 **use formulas for finding the perimeter of a rectangle, triangle, square**

STRATEGY 1: Say, "Daryl Strawberry of the N. Y. Mets hit a homerun in his last game. He ran around the bases. How far did he run?"

Draw on the chalkboard a baseball "diamond".

e.g.,



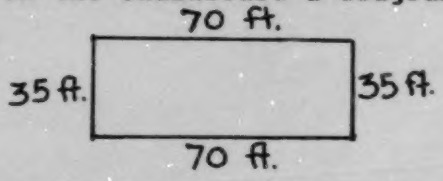
Say that the "diamond" is actually a square, and that we need to find the perimeter of the square to answer the question of the problem, e.g., 90 + 90 + 90 + 90 = 360 ft. Ask the students if there is a shortcut to adding 90 four times. (90+90+90+90 = 4x90) Say that to find the perimeter of a square we multiply the measure of a side by four or P = 4S.

Ask the students to find the perimeter of a square whose side is 8". (P = 4S = 4x8 = 32) Repeat this procedure with other squares.

Say, "A school basketball court measures 70 ft. in length and 35 ft. in width. If you walked around the court, what distance would you have covered?"

Draw on the chalkboard a diagram of the court.

e.g.,



(35+35+70+70 = 210 ft.) Ask if there is any shortcut in the above addition. (Instead of adding the 4 lengths we can multiply the length by 2, and likewise we can multiply a width by 2 and add both

7.10.04

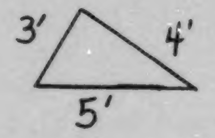
STRATEGY 1: cont'd

products or P = (2xl) + (2xw) or P = 2l + 2w.

Ask the students to find the perimeter of a rectangle whose length is 9" and whose width is 5". (P = 2l + 2w = 2x9 + 2x5 = 18 + 10 = 28) Repeat this procedure using other rectangles.

Ask the students to find the perimeter of a triangle whose sides measure 3', 4' and 5'.

e.g.,



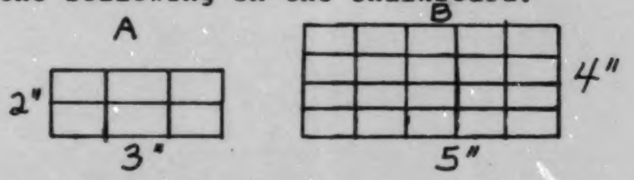
(3'+4'+5' = 12')

Say that the method for finding the perimeter of a triangle can be represented by the formula P = a+b+c. Ask the students to find the perimeter of several triangles.

7.10.04.02 **find the area of polygons by using the appropriate formulas**

STRATEGY 1: Draw the following on the chalkboard.

e.g.,



Ask which rectangle is larger. (B) Say that a measure of the flat space occupied by a plane figure is called the area. Figure B has more area than Figure A. Ask how much greater an area B has than A. (We need to have a unit of measure for area to answer that question.) Say, "What does each of the 'boxes' in the figures measure? (one square inch) How many square inches are contained in each figure? (A has 6 sq. in., B has 20 sq. in.)

STRATEGY 1: cont'd

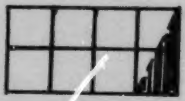
The area of figure A is 6 square inches, and the area of figure B is 20 square inches. We measure area in square units (square inches, square feet, square yards)."

Ask how we can find the area of the above rectangles without counting the squares. (We can multiply the measures of the length by the width.) Say that in general, for any rectangle, the formula for finding the area is A = lxw or A = lw.

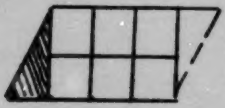
Ask the students to find the area of a rectangle whose length is 6' and whose width is 4'. (A = lw = 6x4 = 24 square feet.)

Say that since a square is a rectangle with length equal to width, we multiply side times side to find the area or A = sxs or A = s^2.

Ask the students to find the area of a square whose side measures 9 cm. (A = s^2 = 9^2 = 9x9 = 81 sq. cm)



On a rectangular sheet of paper draw and elicit that the area is 8 square inches. Cut along the dotted line and attach the triangular section to the left side of the rectangle as follows.



Say that we now have a parallelogram which has the same area as the original rectangle. Ask how the length and width of the rectangle compare to the dimensions of the parallelogram. (The length correlates to the base (bottom side) of the parallelogram and the width correlates to the height (right angle distance from bottom to top) of the parallelogram.)

362

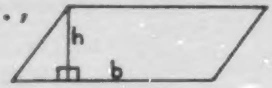
136

7.10.04

STRATEGY 1: cont'd

Draw on the chalkboard the following parallelogram.

e.g.,



A = b x h or A = bh

h = height

b = base

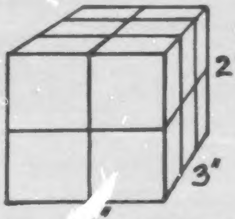
Ask the students to find the area of the parallelogram, drawn as follows, where the base is twelve inches and the height is eight inches. (A = bh = 12x8 = 96 sq. inches)

Repeat this procedure for the area of a triangle.

7.10.04.03 **compute the volume of a rectangular prism and a cube in standard units of measure using the formulas, V = LxWxH and V = BxH**

STRATEGY 1: Draw the following on the chalkboard.

e.g.,



Say that this is a rectangular prism measuring 2" by 3" by 2". Ask how much space the figure occupies. Say that we refer to the measure of space that a solid figure occupies as the volume of the figure.

Ask how we can measure volume. (We need a unit of measure which measures the occupation of space and such a measure is a cubic inch.)

Ask how many cubic inches can be counted in the figure. (12 cubic inches) Say that we can multiply 2 by 2 by 3 = 2x2x3 to get 12 without counting or V = lxwxh or V = lwh.

7.10.04

STRATEGY 1: cont'd

Ask the students to find the volume of a rectangular prism whose dimensions are 8', 5' and 4'. (V = lwh = 8x5x4 = 160 cubic feet)

Repeat this procedure to find the volume of a cube.

STRATEGY 2:

Adapt Strategy 1 used in P.O. 7.10.04.03 using cube blocks to measure the volume of a rectangular prism.

7.10.04.04 **solve verbal problems related to skills learned in this module**

STRATEGY 1:

Write the following on the chalkboard.

> For practice, members of a swimming team swim laps around the perimeter of their school's rectangular swimming pool which measures 60 feet in length and 15 feet in length. How many feet each lap do they swim?

Ask the students how they would solve this problem. (P = 2l + 2w = 2x60 + 2x15 = 150 ft) Ask a student to come to the chalkboard to solve the problem.

Repeat this procedure using other problems.

STRATEGY 2:

Write the following on the chalkboard.

> A rectangular container measuring 3" by 4" by 7" contains cooking oil. Another container of cooking oil measures 2" by 5" by 8". Which container holds more oil?"

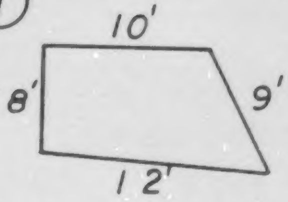
Ask the students how they would solve this problem. (Find both volumes and subtract.) Ask the students to solve the problem in their notebooks while one student works on the chalkboard. Repeat this procedure using other problems.

P.O. 7.10.04.01

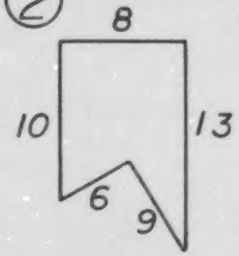
NAME:______________________________ CLASS:__________ DATE:________

Find the perimeter of each of the figures below.

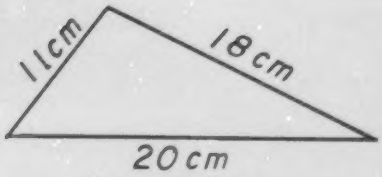
①



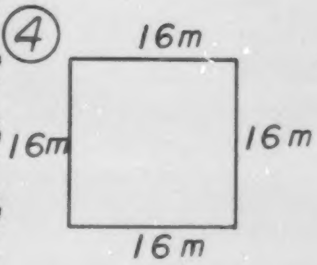
②



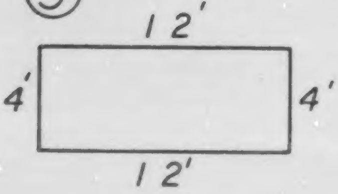
③


11cm
18cm
20cm

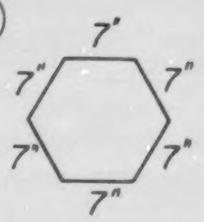
④


16m
16m
16m
16m

⑤

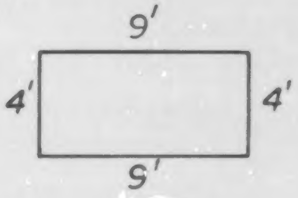


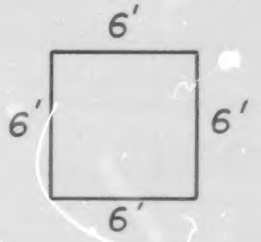
⑥



Which of these figures has the greater perimeter?

⑦





*⑧

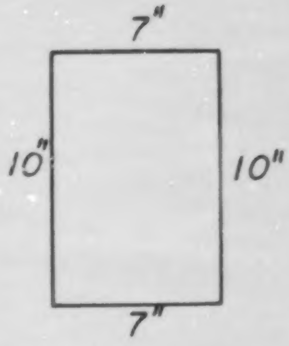
If the length and width of a rectangle is doubled, is the perimeter doubled?

P.O. 7.10.04.02

NAME:______________________________CLASS:__________DATE:________

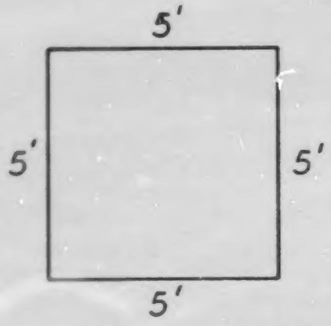
Find the area of each figure.

①



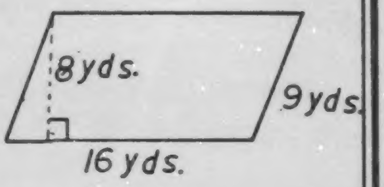
A = __________

②



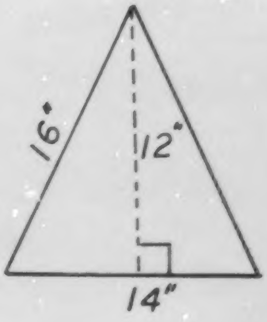
A = __________

③


8 yds.
9 yds.
16 yds.

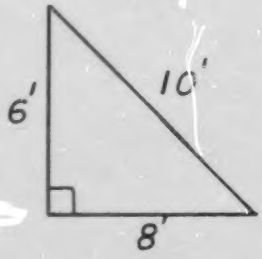
A = __________

④



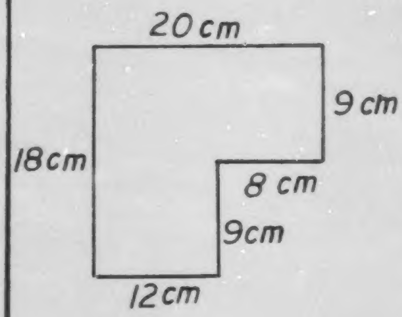
A = __________

⑤



A = __________

*⑥


20 cm
9 cm
18 cm
8 cm
9 cm
12 cm

A = __________

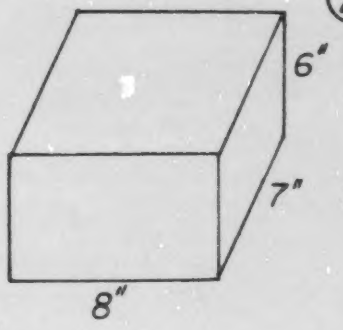
*⑦ If the length and width of a rectangle are doubled, is the area doubled?

P.O. 7.10.04.03

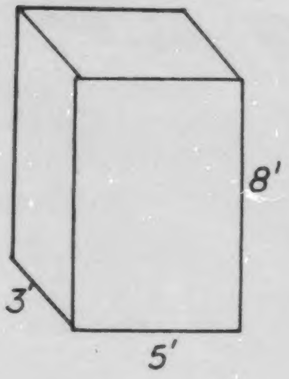
NAME: ______________________ CLASS: __________ DATE: ________

Find the volume of each figure.

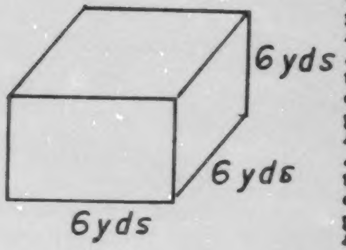
①



②



③


6yds
6yds
6yds

④ Find the volume of a cube whose edge is 4cm

*⑤ If the dimensions of a rectangular solid are doubled, is the volume doubled?

P.O. 7.10.04.04

NAME:______________________________ CLASS:__________ DATE:________

Solve each problem.

1. The Pentagon building in Washington, D.C. measures 921 feet on each side. What distance would you cover if you walked around the outside?

2. How many feet of fencing would be required to enclose a rectangular lot measuring 90' by 120'?

3. A farmer has enough fertilizer to fertilize 10,000 square feet of field. Is this enough to fertilize a field measuring 85' by 105'?

4. An oil tank which is shaped as a rectangular prism measures 10' by 8' by 3'. The tank is to be filled with oil that sells for $4.50 per cubic foot. What will the cost of filling the tank be?

5. A cubic area is to be air conditioned. If each edge of the cube measures 15 yards, how many yards of air conditioning capacity should the air conditioner have?

Unit 39

7.10.05 PERIMETER OF IRREGULAR FIGURES

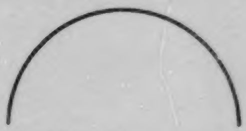
(3 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

7.10.05.01B define and identify a semicircle;

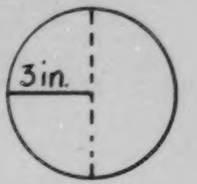
e.g.,



half of a circle = semicircle

7.10.05.02B find the length of a semicircle;

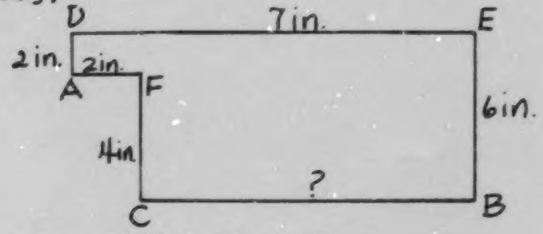
e.g.,



C (semicircle) $= \frac{2\pi r}{2} - \frac{2(3.14)(3)}{2} - 9.42$ in.

7.10.05.03B find missing lengths of irregular figures by applying the properties involving opposite sides of a rectangle;

e.g,



DE = CB + AF

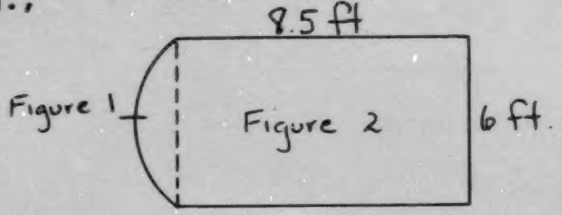
7 in. = 2 in. + ?

7 in. = 2 in. + 5 in.

AF = 5 in.

7.10.05.04B find the perimeter of irregular geometric figures (use π - 3.14).

e.g.,


8.5 ft
Figure 1
Figure 2
6 ft.

Contribution of figure 1 +
Contribution of figure 2 =
Perimeter of Figure

$$\frac{2\pi \quad 3}{2} + 2(8.5) + 6 \simeq$$

$3(3.14) + 17 + 6 \simeq$

$9.42 + 17 + 6 \simeq 32.42$ ft.

7.10.05

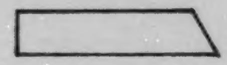
VOCABULARY

TERMS	SYMBOLS
semicircle	
pi	π

GETTING STARTED

Draw three diagrams on the chalkboard and label each.

A)



B)

C)

Ask the students the formula for the perimeter of each figure.

Say, "What is the circumference of a circle if the radius is 5? (3.14 x 2 x 5) What is the perimeter of a rectangle whose length is 9 ft. and whose width is 6 ft.? (2 x 9 + 2 x 6 = 30 ft.) Find the perimeter of a square whose side is 3 meters." (4 x 3 = 12 meters)

Repeat this procedure using other lengths, widths and radii.

Ask the students to state the formula for the circumference of a circle, the perimeter of a rectangle and a square, and write each on the chalkboard. Draw an 18 by 9 inch rectangle to scale on the chalkboard. Ask a student to calculate the perimeter using the formula and ask another student to go to the chalkboard and measure the perimeter. Say, "Are they the same?"

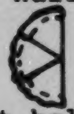
Repeat this procedure using various shapes and lengths.

7.10.05

7.10.05.01 **define and identify a semicircle**

STRATEGY 1: Using a flannel board, make a pizza composed of six slices out of cardboard and back each piece with sandpaper. Place the slices on the flannel board to make half of a pizza.

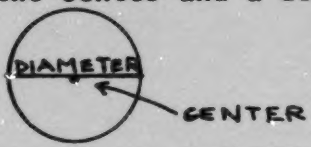
e.g.,



Say that half of a circle is called a semicircle. Ask students to come to the flannel board to rearrange the slices to create various semicircles.

STRATEGY 2: Draw a picture of a circle on the chalkboard and label the center and a diameter.

e.g.,



Say, "The diameter divides a circle into two parts. Each part is called a semicircle." Ask a student to come to the chalkboard and color each semicircle using a different color chalk.

Ask the students to draw a circle and divide it into two semicircles in their notebooks. Ask the students to draw several circles and have the diameter divide the circle in different ways.

7.10.05.02 **find the length of a semicircle**

STRATEGY 1: Draw a circle on the chalkboard and show the measure of the radius. Ask a student how we find the circumference of the circle. (The circumference equals $2\pi r$.) Ask a student to come

STRATEGY 1: cont'd

to the chalkboard and determine the circumference of the circle.

Say, "How would you find the circumference of half ($\frac{1}{2}$) of the circle?" (You would divide the circumference of the whole circle by two.) Write the new formula on the board.
e.g.,

$$C(\text{semicircle}) = \frac{2\pi r}{2}.$$

Draw a circle with a radius of 8 inches on the chalkboard. Ask a student to come to the chalkboard and substitute the numbers in the formula to find the circumference of the circle.
e.g.,

$$C = 2\pi(8)$$
$$C = 25.12$$

Divide the circle in half and ask a student how to determine the circumference of $\frac{1}{2}$ the circle or the semicircle.
e.g.,

$$\frac{25.12}{2} = 12.56$$

Repeat this procedure using various radii, e.g., 3 ft, 6 inches, 2 meters, 4 cm.

STRATEGY 2: Prepare and distribute a compass, a piece of string and a ruler to each student. Ask the students to use a compass to draw a circle in their notebooks while you draw one on the chalkboard. Using a string, measure your circumference on the chalkboard while the students measure their circumferences using the same technique. Measure the length of string with a ruler. After the students have measured the circumference of their circles, draw a diameter through the center of your circle. Measure the circumference of $\frac{1}{2}$ of the circle using the above technique while the students do the same in their notebooks. Say, "Looking at

7.10.05

STRATEGY 2: cont'd

the measures of both the circumference of the circle and the circumference of the semicircle, what can you discover?" (The circumference of the semicircle equals half of the circumference of the circle.)

Write the formula $C \simeq 2\pi r$ on the chalkboard. Say, "If this is the formula for finding the circumference of a circle, what would you do to find the circumference of a semicircle?" (The circumference of a semicircle is half that of a circle so therefore, we must take half of the formula.) Ask a student to come to the chalkboard and write the new formula.

e.g.,

$$C\ (\text{semicircle}) \simeq \frac{2\pi\ r}{2}$$

Draw a circle on the chalkboard with a radius of 6 inches. Ask the students to find the circumference of the circle, and then, of the semicircle.

e.g.,

$C(\text{circle}) \simeq 37.89$ inches

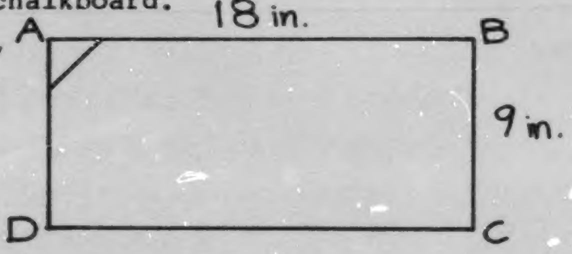
$C(\text{semicircle}) \simeq 18.84$ inches.

Repeat this procedure using various circles.

7.10.05.03 **find the missing lengths of irregular figures by applying the properties involving opposite sides of a rectangle**

STRATEGY 1: Ask the students to state the properties of a rectangle. (Opposite sides are equal, all four angles are right (90°) angles.) Draw a diagram on the chalkboard.

e.g.,

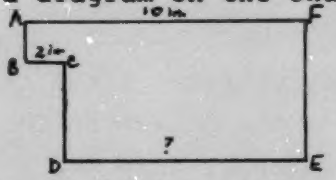


STRATEGY 1:
cont'd

Ask the students if they can find the length of CD. (CD must be equal to AB, therefore, CD = 18 in.) Ask the students if they can find length BD. (BD must be equal to AC, and therefore, BD = 9 inches.)

Draw a diagram on the chalkboard.

e.g.,



Ask the students to look at the drawing and notice that DE is not named in inches. Say, "How might we figure out the length of DE?" (The piece labeled BC is 2 inches and if you were to cut it off, AF would have the length of DE.)

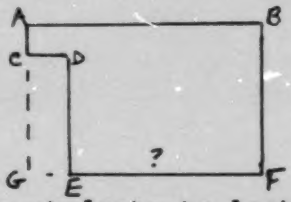
Write the following on the chalkboard.

DE = AF - 2 inches
DE = 10 - 2 inches
DE = 8 inches

Repeat this procedure using various shapes and numbers. All shapes must have a rectangle as their base.

STRATEGY 2: Draw a diagram on the chalkboard.

e.g.,



Ask the students to look at the drawing and notice that EF is not named in inches. Complete the rectangle CDEG on the chalkboard using dotted lines. Say, "Using the properties of a rectangle, what would GE equal?" (GE = CD. If CD = 2 inches, then GE = 2 inches.) Therefore, EF = AB - 2 inches, EF = 10 - 2 inches, EF = 8 inches.

Repeat using various diagrams and lengths. All shapes must have a rectangle as their base.

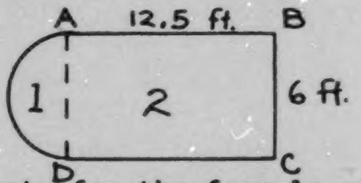
7.10.05

7.10.05.04 **find the perimeter of irregular geometric figures (use π = 3.14)**

STRATEGY 1: Ask the students to state the formulas for finding the perimeter of various shapes, e.g., circle, square, rectangle.

Draw an irregular diagram on the chalkboard. Say that to determine the perimeter for this geometric shape we must divide it into two shapes. Using a dotted line, separate the figure into two shapes. Ask a student to identify the shapes. Elicit that one is a semicircle and the other is a rectangle. Label them figure 1 and figure 2.

e.g.,



Ask the students for the formula of each. Ask a student to come to the chalkboard and fill in the missing sides. Using the numbers given, ask the students to figure out the perimeter. (The circumference for figure 1 is 18.84 ft., and the perimeter for figure 2, 37 ft.) Say that line AD is not part of the original drawing, and therefore, should not be added on to the perimeter. Say, "How would we find the perimeter of the entire figure?" (We add the circumference and the perimeter.) Ask a student to come to the chalkboard and add 37 + 18.84 = 55.84 ft.

Repeat this procedure using various irregular figures.

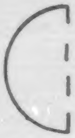
STRATEGY 2: Using a flannel board or pieces of oak tag create a shape.

e.g.,

7.10.05

STRATEGY 2: cont'd

By moving the pre-cut shapes apart, show the students that this shape is actually two shapes put together.



a semi-circle,

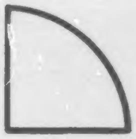
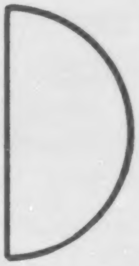
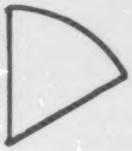
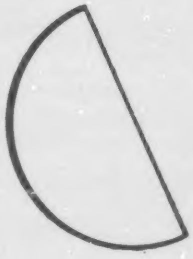
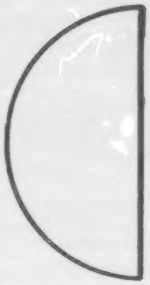
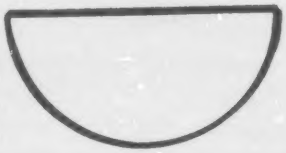
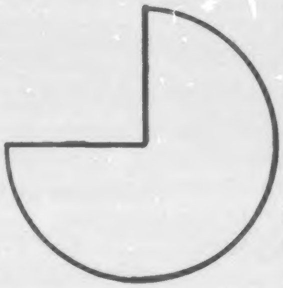
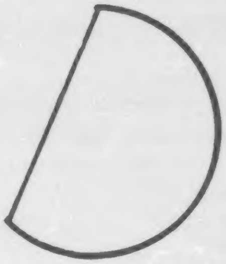
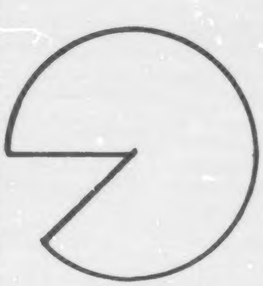
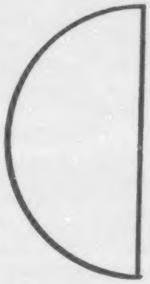
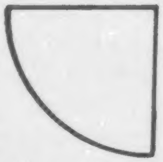
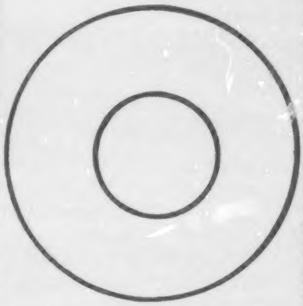
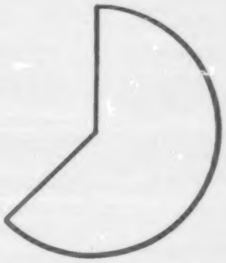
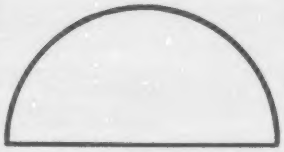
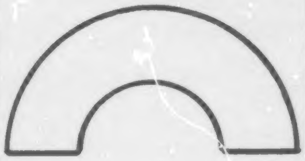
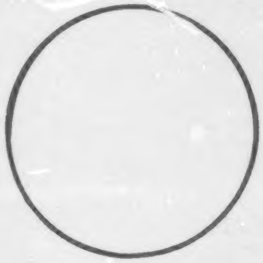
and a rectangle

Say that to find the perimeter of this shape we must add the circumference of the semicircle to the perimeter of the rectangle. Remember the side AD does not get added into the perimeter because it is not part of the shape itself. Write the measures of the sides and ask the students to find the perimeter and add it to the circumference. Repeat this procedure using various irregular shapes.

P.O. 7.10.05.01

NAME:______________________________CLASS:________DATE:________

Darken in the semi-circles.

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P.O. 7.10.05.02

NAME:____CLASS:__________DATE:________

Given the radius of each circle find the circumference of its semi^{2} circle.

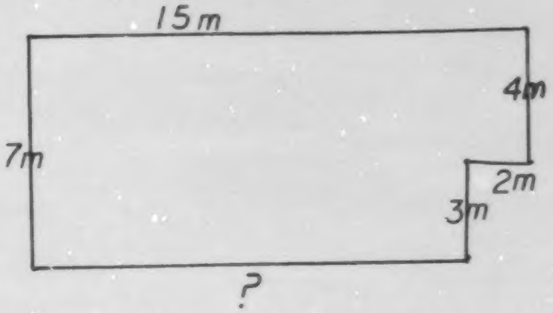
	Radius	Work Space	Circumference of the semi-circle
Example	4 meters	$C = \frac{\pi 2r}{2}$ $C = \frac{25.12}{2}$ $C = 12.56$	12.56m
	12 inches		
	100 cm		
	24 feet		
	32 miles		
	116 yards		
	82 inches		

P.O. 7.10.05.03

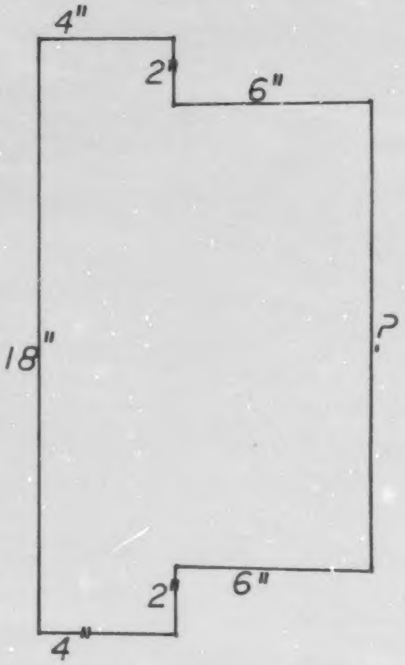
NAME: ______________________ CLASS: __________ DATE: ________

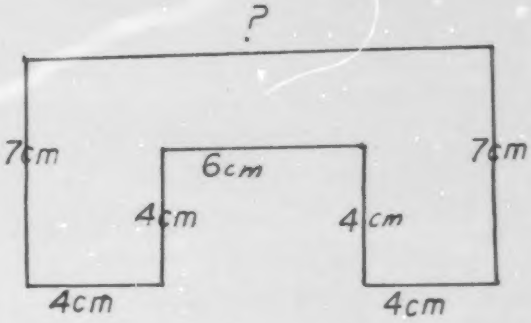
Find the missing side.

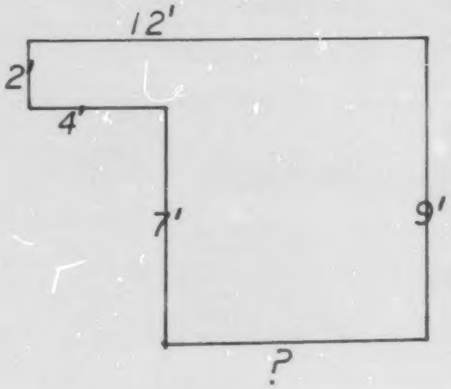
Example:


15m
4m
7m
2m
3m
?

15 m − 2 m = 13 m




?
7cm
6cm
7cm
4cm
4 cm
4cm
4cm



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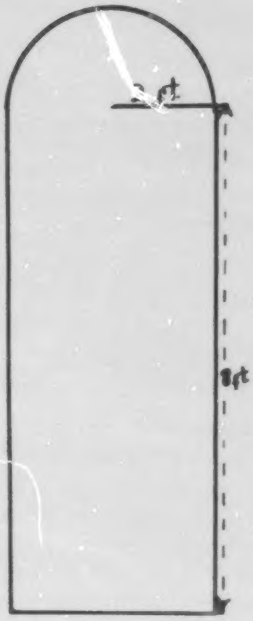
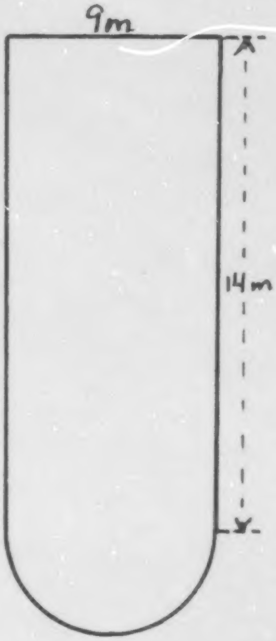
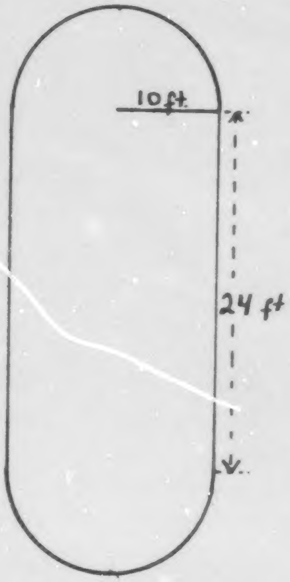
P.O. 7.10.05.04

NAME:______________________________________CLASS:_________DATE:_______

Find the perimeter of each of the geometric shapes.







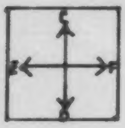
7.09.05 SPATIAL FIGURES

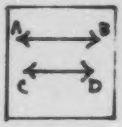
(7 lessons)

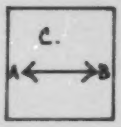
PERFORMANCE OBJECTIVES

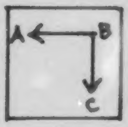
The student will be able to...

7.09.05.01B define a plane as being determined by
(1) two intersecting or parallel lines,
(2) three non-collinear points, or
(3) a line and a point not on the line;
e.g.,

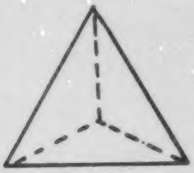








7.09.05.02A identify and count the number of faces, edges, vertices and bases in a given rectangular prism or triangular prism.
e.g.,



number of edges = 6
number of faces = 4
number of bases = 1
number of vertices = 4

VOCABULARY

TERMS

dimensions
plane
polyhedron

GETTING STARTED

Prepare and display the basic geometric solids, including pyramids, cubes, rectangular solids and prisms. Ask the students to identify the features of each figure, and define a point, line and plane. Discuss the relationships between a point, a line and a plane, by asking these questions.
e.g.,

1) Can a point be on a plane?
2) Can a line be on a plane?
3) Can a point and a line be on a plane?
4) Can two parallel lines be on a plane?

Repeat this line of questioning with other possibilities.

Ask the class if they have heard of the word dimension. Say that in our world everything exists in three dimensions: length, width and depth. Ask the students to name some of the things in our everyday world that have three dimensions, e.g., people, desk, pencil, paper...

Say, "Certain things in our world only have two dimensions, length and width. Can you name some?" (Pictures on paper, movies on TV and all things that are flat are two dimensional.)

Draw a picture of a rectangle on the chalkboard and ask a student if it is 2 or 3 dimensional.

Repeat this procedure using other familiar objects.

7.09.05

7.09.05.01 **define a plane as being determined by (1) two intersecting or parallel lines, (2) three non-collinear points, or (3) a line and a point not on the line**

STRATEGY 1: Prepare one large sheet of cardboard and several pencils. Say, "I would like to build a table, but I'm not sure how to keep it up." Hold up one pencil point. Ask if the cardboard would stay up on just one pencil point. Hold up three pencil points in a row. Show that it still will not stay up. (In order to hold up the piece of cardboard we need 3 points not in a row.)

Hold up the side of a pencil, horizontal to the floor. Ask if the cardboard will stay up on the one pencil. Ask what would happen if we added another pencil point. (It would stay up with one flat pencil and also a pencil point.)

Hold up two pencils horizontal to the floor and perpendicular to each other. Ask if this combination would work. (Two flat pencils that intersect each other will work.) Ask what else will work. (Two parallel pencils horizontal to the floor will also work.)

Ask the class to imagine that the piece of cardboard was a plane, the pencil point was a point, and that the pencil held horizontal to the floor was a line. Say, "What can we conclude from this demonstration?" Elicit that a plane can be formed with the following:

a) three non collinear points;
b) a line and a point not on the line;
c) two intersecting lines;
d) two parallel lines.

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STRATEGY 2: Ask if anybody has a chair at home that wobbles. Say that a chair with four legs wobbles because one of the legs is either too large or too small. Ask if a stool with three legs can wobble. (A three legged stool cannot wobble because all three of its legs must always be on the floor at the same time.) To prove this you may want to use a plastic lid with three pipe cleaners of various lengths. Say that the reason a three legged stool always stands steady is because its three legs form a plane or flat surface. Say that there are four conditions that form a plane:

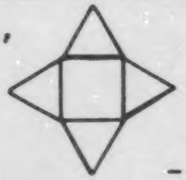
1) 3 noncolinear points;
2) 2 intersecting lines;
3) 2 parallel lines;
4) a line and a point not on that line.

Ask the students to demonstrate the possibilities of making wobbleless furniture using the above ideas.

7.09.05.02 **identify and count the number of faces, edges, vertices and bases in a given rectangular prism or triangular prism**

STRATEGY 1: Ask the students if they've ever heard about the pyramids of Egypt. Say, "Who can describe them to us?" (A pyramid is made up of triangles that meet at a point.) Say that the number of triangles in a pyramid is determined by the shape of the base. Say that the pyramids in Egypt have a square or rectangular base, and since a square or rectangle has four sides, we need four triangles to form a pyramid. Demonstrate by drawing the following diagram on the chalkboard.

e.g.,



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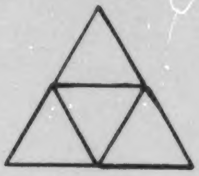
STRATEGY 1: cont'd

Demonstrate that if you fold up the four triangles, you will form a pyramid. Ask the students to identify the base, faces, edges and vertices of this pyramid.

Draw a picture of a triangle on the chalkboard. Say, "What if this was the base of a pyramid? How many triangles would we need to complete the pyramid?" (We would need three more triangles.) Demonstrate that when we fold up the three triangles, we also wind up with a pyramid.

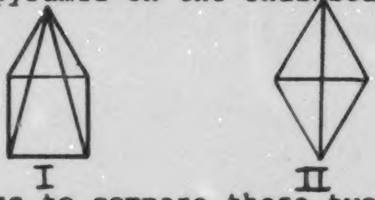
e.g.,



Ask the students to identify the base, faces, edges and vertices of this pyramid, and compare the data between a rectangular pyramid and a triangular pyramid. (Rectangular pyramid has five faces and eight edges while a triangle pyramid has four faces and six edges.)

STRATEGY 2: Draw a picture of a rectangular pyramid and a triangular pyramid on the chalkboard.

e.g.,

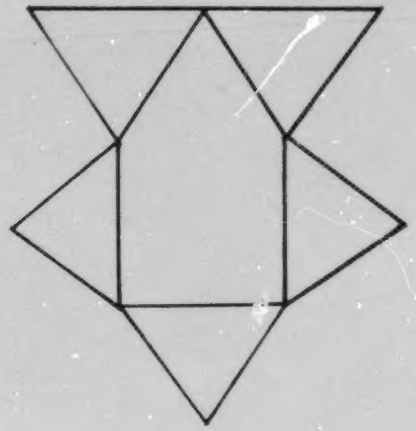


Ask the class to compare these two pyramids and to write down the features of each one. (Pyramid I has a square for a base, four triangles that meet at the top vertex, five faces, eight edges and five vertices. Pyramid II has a triangle for a base, three triangles that meet at the top vertex, four faces, six edges and four vertices.)

STRATEGY 2: cont'd

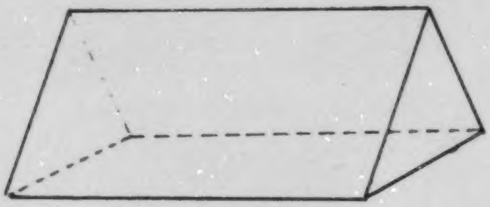
Ask the class if they could imagine a pyramid with a pentagon as a base. Say, "How many faces, edges and vertices would it have?" Draw this figure on the chalkboard and discuss it.

e.g.,

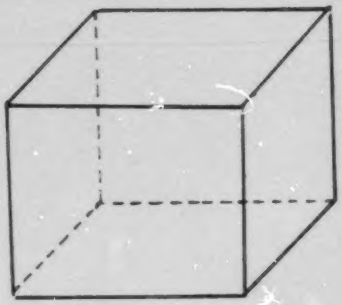


P.O. 7.09.05.02

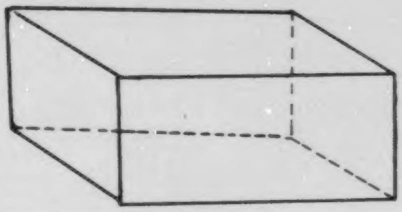
NAME:____________________ CLASS:________ DATE:________



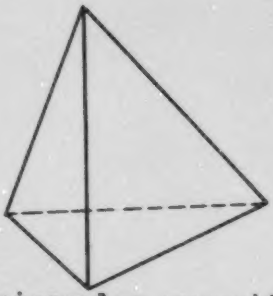
Triangle prism



Cube



Rectangular prism



Triangular pyramid

Count the faces, vertices, and edges for each shape and complete the table.

Polyhedron	Faces	Vertices	Edges
Triangular prism			
Cube			
Rectangular prism			
Triangular pyramid			

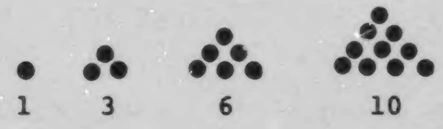
7.01.04 TRIANGULAR NUMBERS, SQUARE PATTERN

(2 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

*7.01.04.01A identify and construct triangular numbers; e.g.,

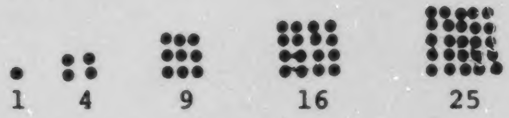


*7.01.04.02A identify the patterns for triangular numbers; e.g.,

The sum of consecutive counting numbers is a triangular number.

$1 + 2 + 3 = 6$

*7.01.04.03A construct a square pattern; e.g.,



*7.01.04.04A identify patterns for squares. e.g.,

$1^2 + 2^2 + 3^2 + 4^2 ...$

$1 + 4 + 9 + 16 ...$

VOCABULARY

TERMS

triangular numbers
square pattern
square numbers

GETTING STARTED

Play the game "Know the Rule." Ask a student to say any number, e.g., 8. Respond by saying, "12." Ask another student to say a number, e.g., 14. Respond, "18." Continue asking two more students for numbers. Then say, "What is the rule?" (Four was added to each number.)

Repeat the game using another pattern.

Draw a triangle on the chalkboard. Say, "What are the properties of a triangle?" Write the properties on the chalkboard. Draw a rectangle on the chalkboard. Ask the students to name the properties of a rectangle. Write the properties on the chalkboard.

Repeat this procedure for squares.

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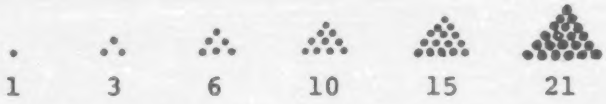
7.01.04

7.01.04.01 **define, identify and construct triangular numbers**

STRATEGY 1: Prepare sets of 30 circles, one set for every two students. Divide the class into groups of two and distribute a set of circles to each pair of students. Say, "We are going to use these circles to form triangles. Start with 1 circle and work with the circles in consecutive numerical order. e.g., 2 circles, 3 circles, 4 circles... 30 circles. A triangular number is one whose circles form a triangular pattern. See which number of circles form a triangle and record and draw the circle pattern in your notebooks."

e.g.,

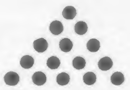


Give the students about 10 to 15 minutes to discover the triangular numbers. Ask a student to come to the chalkboard and draw the first triangular number.

Continue asking students to come to the chalkboard to draw the triangular numbers up to 28.

STRATEGY 2: Ask several students to come to the chalkboard and construct a triangular shape by drawing circles. Say, "How many circles were used?"

e.g.,



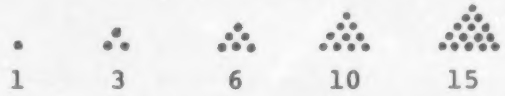
List on the chalkboard several larger numbers used to construct triangles, e.g., 28, 36. Ask other students to come to the chalkboard to use larger numbers of circles to construct the triangles for these numbers. Say that the number of circles which form a triangular pattern are called triangular numbers. Direct the remaining students to work in their notebooks.

7.01.04

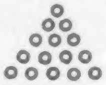
7.01.04.02 identify patterns for triangular numbers

STRATEGY 1: Prepare cards with pictures of several triangular patterns on them. Form groups of five students and distribute cards to each group. Ask the students to look at the numbers and find any numerical patterns. Ask one student in each group to act as the recorder, writing down each pattern found. e.g., the sum of consecutive numbers starting with 1 gives a triangular number, the sum of 2 consecutive triangular numbers yields another triangular number... After allowing approximately 10 minutes for group exploration, ask each recorder to read the group's findings. Write the findings on the chalkboard.

STRATEGY 2: Display

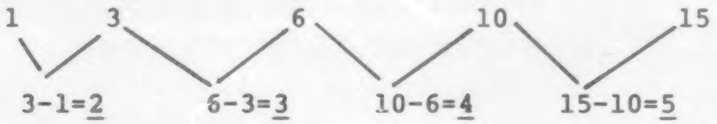


on the chalkboard. Say, "How would we find the next number? Do you see a pattern to the construction of the triangular numbers?" (The numbers are built by the addition of consecutive numbers of circles.)
e.g.,



5 + 4 + 3 + 2 + 1 = 15.

Say, "Do you see any other patterns?" Write all patterns on the chalkboard. (We add one to the previous difference between triangular numbers to get the next triangular number.)
e.g.,



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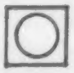
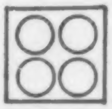
7.01.04

7.01.04.03 construct a square pattern

STRATEGY 1: Prepare 40 circles or use magnetized circles. Tape one circle to the chalkboard. Say, "By drawing a square around this circle, we say that we have constructed a square pattern with the number one. If we add three circles to this one and then draw a square around them, we will have another square pattern."

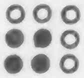
e.g.,

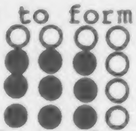




Ask how many circles we now have. Continue adding groups of circles to form square patterns.

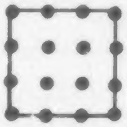
e.g.,





STRATEGY 2: Distribute to each student a worksheet with dots on it in rows and columns. Say, "We are going to use these dots to construct different size squares. Connect the dots to form a square, then count all of the dots inside the square and on the edge and write the number in the square. Be sure to make different sized squares."

e.g.,



Allow several students to come to the chalkboard to draw their square patterns on a larger scaled version of the worksheet. List the patterns in ascending numerical order.

7.01.04

7.01.04.04 **identify patterns for squares**

STRATEGY 1: Prepare and display a chart with square numbers from 1 to 49. Ask the students to look at the numbers and determine any number patterns, e.g., each square number is the square of a counting number ($1^2 = 1$, $2^2 = 4$, $3^2 = 9$, $4^2 = 16$). Allow the students to work independently for several minutes. Write the patterns that the students discover on the chalkboard.

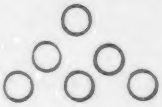
STRATEGY 2: Play "Pattern Discovery". Draw square patterns on the chalkboard. Say to the students that you will give them 10 minutes to find and write all the patterns that they can in square numbers. After 10 minutes ask a student to stand and read the list of patterns. For each valid pattern students get 1 point. The student(s) with the most points wins.

P.O. 7.01.04.01-02

NAME: ______________________________ CLASS: __________ DATE: ________

I. Count the number of circles in each triangular pattern.
II. See directions on sheet.

I.



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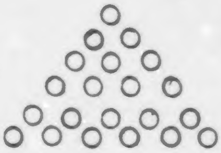

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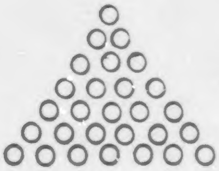

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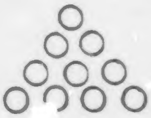

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II. We can express a triangular number as the sum of consecutive numbers, e.g. 1+2+3=6. Express each number below as the sum of consecutive counting numbers.

1. 28

2. 21

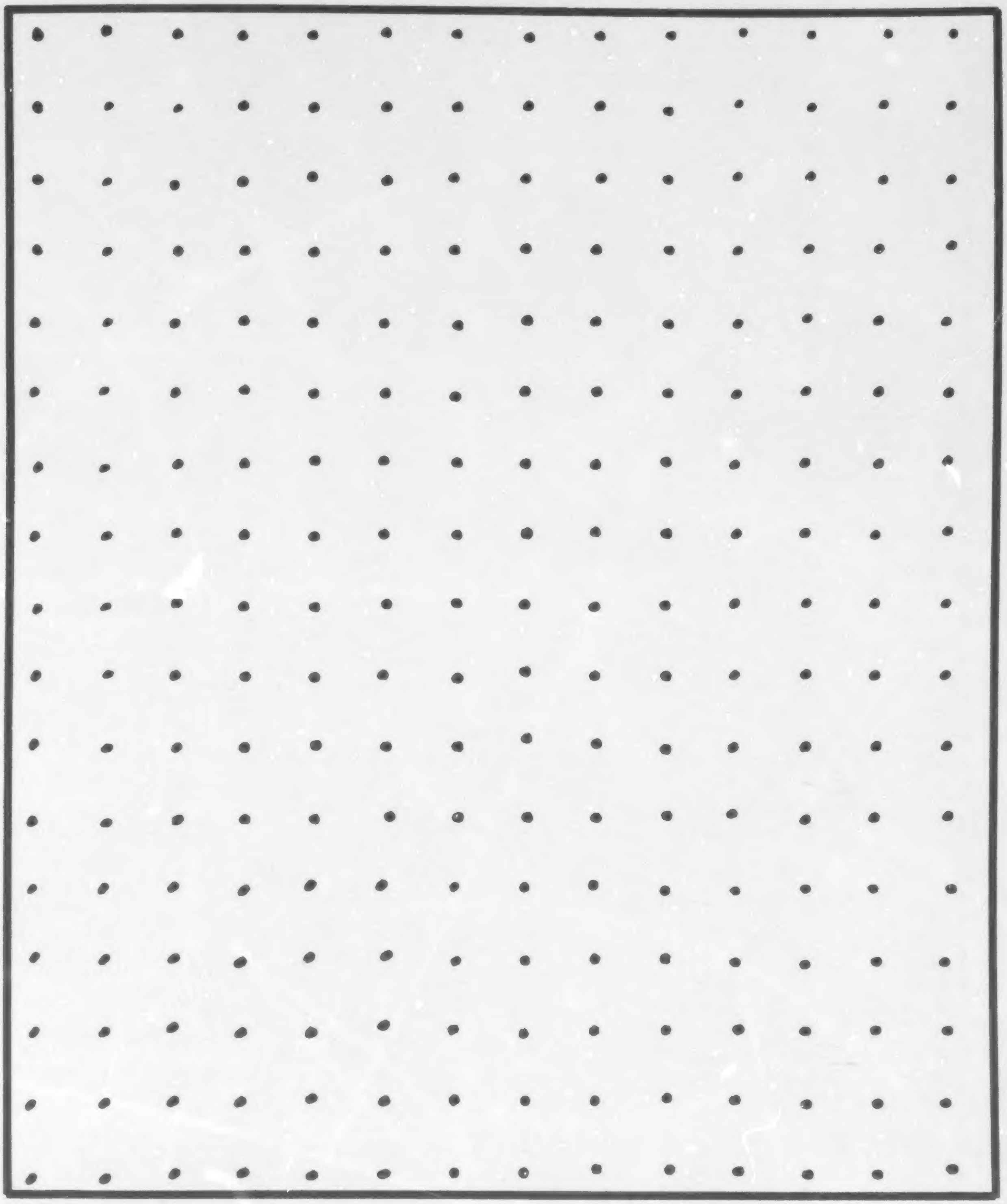
3. 45

4. 36

P.O. 7.01.04.03-04

NAME: ________________________________ CLASS: __________ DATE: ________

Connect the dots to form squares of different sizes.
Write the number of dots inside of each square.



Unit 42

7.01.05 PASCAL'S TRIANGLE

(1-2 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

*7.01.05.01A construct Pascal's Triangle;
e.g.,

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

*7.01.05.02A identify patterns in addition of rows;
e.g.,
4th row: $1 + 3 + 3 + 1 = 8$ or 2^3
5th row: $1 + 4 + 6 + 4 + 1 = 16$ or 2^4

*7.01.05.03A identify patterns in Pascal's Triangle.
e.g.,
All rows are palindromic.

7.01.05

VOCABULARY

TERMS

Pascal's triangle
exponent
exponential notation

GETTING STARTED

Prepare 20 cards with a number sequence on each. Make sure two numbers have been left out of each sequence, e.g., 4,7,___,13,16,___ or 41,39,37,___,33,___. Say, "We are going to play a sequence game." Divide the class into two teams. State the rules of the game.

1. The team that goes first will have a member choose a sequence card.
2. That student will write the sequence on the chalkboard and fill in the blanks in 30 seconds. If the student does not complete the sequence in the alloted time, the other team will have a chance to complete the blanks.
3. The team that completes the blanks will get 1 point. The first team which gets 10 points wins the game.

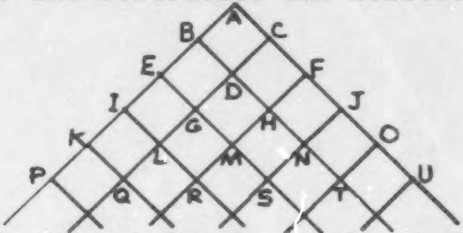
Choose which team goes first and begin to play.

Write 7,10,13,16,19,___,___,___ on the chalkboard. Ask a student to come to the chalkboard and fill in the three blanks. Say, "What is the pattern in this series of numbers? (Add 3) Do all number sequences have the same pattern? How do we discover the pattern in a number sequence?" (We study the first few numbers and determine their relationship to each other.) Continue this procedure using 4,5,9,14,___,___,___, and 10,11,9,___, and 10,8,___,___,___, and 1000,999,997,994,990,___,___,___.

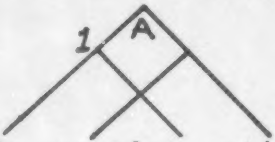
7.01.05

7.01.05.01 **construct Pascal's Triangle**

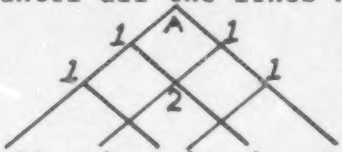
STRATEGY 1: Prepare and distribute the following rexograph sheet.



Draw the same triangle on the chalkboard without the letters. Say, "This is a pinball machine. A ball starts at A. How many different ways can the ball go to get from A to B?" Write 1 on the chalkboard where B is located on the rexograph sheet.

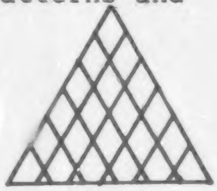


Continue this procedure until all the 1 lines have a number. Say, "How many ways can the ball go to get from A to D?" Write 2 on the chalkboard where D is located on the rexograph sheet. Continue this procedure until all the lines have a number.



Say, "We call this triangle Pascal's Triangle. We can use this triangle to discover many patterns and sequences."

STRATEGY 2: Draw a triangle on the chalkboard, e.g.,



Say, "This is our town and these are the blocks in our town. The Transportation Commissioner is asking us to design a bus route for the town. In order to do this, we must find out how many different routes there are. If we start at the top, how many ways are there to get to the first

STRATEGY 2: cont'd

block? Since there is only 1 route, write a 1 at the x." Continue until you have discovered the numbers of all the routes and the triangle looks like this.

e.g.,

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

Say, "This bus route chart looks exactly like a famous mathematical figure called Pascal's Triangle."

7.01.05.02 **identify patterns in addition of rows**

STRATEGY 1: Prepare and distribute a sheet on which Pascal's Triangle is drawn. Say, "Add the numbers across each row and put the sum outside of the triangle.

e.g.,

Row Number		Sums
1	1	1
2	1 1	2
3	1 2 1	4
4	1 3 3 1	8
5	1 4 6 4 1	16

Look at all the sums. Do you see any pattern to these numbers? e.g., $2 = 2^1$, $4 = 2^2$, $8 = 2^3$, ... Look at the sum at the end of each row, e.g, row 5, sum of all numbers in the row = 16, 16 equals 2^3 (3 = number of the row). What would the sum of the numbers in row 8 be?" (2^8 or 256)

STRATEGY 2: Draw Pascal's Triangle on the chalkboard. Ask a student to come to the chalkboard and add the numbers in the second row of the triangle. Write the sum next to the triangle.

STRATEGY 2: cont'd

Continue this procedure until all the numbers in each row have been added. Ask the students to look at the sums and see if they can find any patterns in the sums, e.g., multiply each number by 2. Say, "Do we know any way to show repeated multiplication? (exponents) How can we write 2 in exponential notation?" Write 2^1 on the chalkboard next to the sum. Repeat this procedure for $4(2^2)$, $8(2^3)$, ... Ask if the students see any pattern between the row number and the exponent. Write sum = 2^{row} on the chalkboard. Say, "What would the sum of the 9th row be in exponential notation?" (2^9) Repeat the question using the 18th row, the 27th row, ...

7.01.05.03 **identify patterns in Pascal's Triangle**

STRATEGY 1: Draw Pascal's Triangle on the chalkboard. Ask the students to copy the triangle in their notebooks. Divide the students into groups of 4 or 5. Say, "I would like each group to study the triangle and see if you can find at least one pattern in the triangle. Write the rules for the patterns in your notebooks." Allow the students to investigate the triangle and write the patterns for about 10 minutes. Ask each group to select a spokesperson to report the group's findings. Allow each spokesperson to report to the class and write each pattern found on the chalkboard, e.g., all rows are palindromic, the second diagonal has a pattern (1,2,3,4,...) of + 1, the first and last numbers are always 1, every odd row has a double number in the middle,...

STRATEGY 2: Prepare enough cards with Pascal's Triangle drawn on each for half of the students in your class. Pair the students and distribute a card to each pair. Ask the students to investigate the triangle for five minutes to discover some pattern in the triangle. When the investigative time has expired, ask each pair to state what they found. Ask a student to come to the chalkboard to record the findings.

P.O. 7.01.05.01-.03

NAME: ______________________ CLASS: ________ DATE: ________

I. Fill in each ☐.
II. Fill in each blank.

I.

Pascal's Triangle

Row Number		Row Sum
0	☐	______
1	☐ ☐	______
2	☐ ☐ ☐	______
3	☐ ☐ ☐ ☐	______
4	☐ ☐ ☐ ☐ ☐	______
5	☐ ☐ ☐ ☐ ☐ ☐	______
6	☐ ☐ ☐ ☐ ☐ ☐ ☐	______
7	☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐	______

II.

1. The sum of row 8 would be 2 or ______.

2. The sum of row 15 would be 2 or ______.

3. Complete these rows of Pascal's Triangle.

Row Number

8 1 8 28 56 70 __ __ __ __

9 1 9 36 84 126 126 __ __ __ __

10 1 10 45 120 252 __ __ __ __

7.01.06 FIBONACCI NUMBERS

(1-2 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

7.01.06.01B	list the first 10 numbers in the Fibonacci sequence; e.g., 1, 1, 2, 3, 5, 8, 13, 21, 34, 55
*7.01.06.02A	identify patterns with Fibonacci numbers. e.g., The sum of the first 2 numbers equals the third and so on. 1 + 1 = 2, 1 + 2 = 3, 2 + 3 = 5, etc.

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VOCABULARY

TERMS

Fibonacci number
pattern
sequence

GETTING STARTED

Write a number sequence on the chalkboard, e.g., (1, 3, 6, 9, 12,...), (15, 18, 21, 24, 27...) Ask a student to come to the chalkboard and write the next five numbers in the sequence. Say, "What do we call this line of numbers? (pattern or sequence) Here are some other patterns." Write them on the chalkboard.

e.g.,

(1) 1, 5, 9, 13,... (17, 21, 25, 29, 33...)
(2) 1, 2, 4, 7, 11,... (16, 22, 29, 37, 46)
(3) 1, 4, 7, 10,... (13, 16, 19, 22, 25)

Select three different students to name the next five terms in each pattern. Ask the students to give a rule for choosing the next term in sequence, (1) (n+4). Ask another student to give a rule for choosing the next number in sequence, (2) n+1, n+2, n+3, n+4, ... Continue for sequence (3). Say, "We define a sequence as a number pattern with a rule for continuing the sequence."

Play a sequence game. Divide the class into two teams, A and B. Choose a captain for each team. Explain the rules of the game.

1. Each team will submit at least ten number sequences to the teacher before the game begins.
2. Set a time limit for each team member in which to answer each question, e.g., 25 seconds, 1 minute.
3. Any member who exceeds the time limit forfeits his/her turn and the other team gets a point.
4. Each correct answer receives one point.
5. The team to reach six first wins the game.

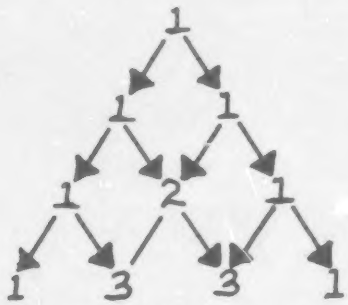
GETTING STARTED cont'd

Ask each team to spend several minutes writing number sequences to be submitted. Ask the team captains to choose which team goes first. The first member of the team which goes first will come to the chalkboard and write the first three numbers of one of the patterns submitted to the teacher. The first member of the other team will come to the chalkboard and write the next five numbers in the sequence. If the five numbers are correct, the team completing the sequence gets a point. The game will continue, each team taking turns writing patterns or completing patterns, until one team reaches six points.

7.01.06

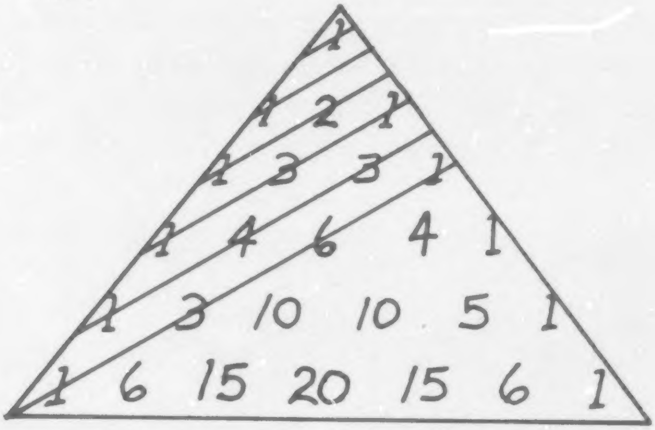
7.01.06.01 list the first 10 numbers in the Fibonacci sequence

STRATEGY 1: Prepare rexograph sheets with Pascal's Triangle. Write four rows of Pascal's Triangle on the chalkboard. Ask a student to fill in the fifth row. Continue to do this until ten rows in the triangle are complete.



Say, "We can find another famous sequence of numbers by using Pascal's Triangle." Distribute the rexograph sheet. Ask students to draw diagonal lines on the triangle. Demonstrate on a display model.

e.g.,



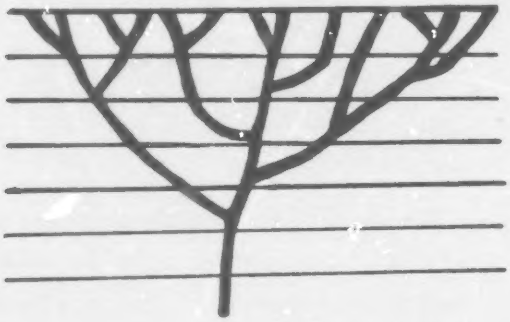
STRATEGY 1: cont'd

Ask the students to add the numbers across the diagonal rows.

Row 1 - 1
Row 2 - 1
Row 3 - 2
Row 4 - 3
Row 5 - 5 ...
Row 10 - 55

Say, "These numbers are called Fibonacci numbers and are named after the man who recognized them."

STRATEGY 2: Distribute this picture to each student.



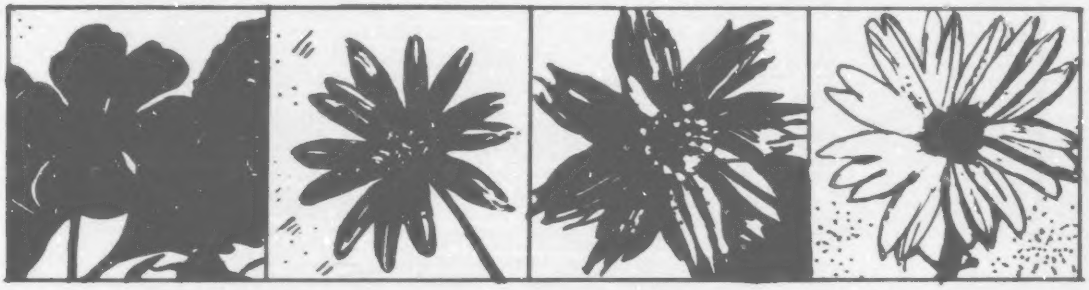
Say, "Look at the bottom of this picture. When the figure crosses the first horizontal line, how many branches does it have? (1) When it crosses the second line? (1) Third line? (2) ...Seventh line? (13)" Write each number on the chalkboard. Say that these numbers are called Fibonacci numbers. They were named for Leonardo of Pisa (1175-1230) also known as Fibonacci. Ask the students if they see a way to use the numbers to find the next three numbers. (8 + 13 = 21, 13 + 21 = 34, 21 + 34 = 55)

7.01.06

7.01.06.02 identify patterns with Fibonacci numbers

STRATEGY 1: Write the first 10 Fibonacci numbers on the chalkboard, e.g., 1, 1, 2, 3, 5, 8, 13, 21, 34, 55. Say, "How can we find all of the Fibonacci numbers less than 100?" (Add the last two numbers to get the next number.) Write them on the chalkboard. Ask the students to use the numbers to experiment and find other patterns, e.g., the sum of any two consecutive odd numbers is a Fibonacci number, the Greater Common Denominator of any two Fibonacci numbers is a Fibonacci number, ...

STRATEGY 2: Prepare and distribute xerox copies of this picture.



Say, "Can we find Fibonacci numbers in these flowers? Which numbers are represented?" Say that there are many patterns in Fibonacci numbers. Divide the students into four groups. Ask one group to use a calculator and find the next five Fibonacci numbers after 317, 81 and 514, 229. Write them on the chalkboard. Ask another group to divide a term of the series by the term immediately preceding it, e.g., $\frac{5}{3}$, $\frac{13}{8}$, ... and write down their answers. Ask the third group to choose two numbers in the series and find the GCD. Ask the fourth group to sum any group of numbers in the series and add one to the result. Ask the students to find their own patterns.

P.O. 7.01.06.01-02

NAME: ______________________________ CLASS: __________ DATE: ________

Fill in each blank.

I. Complete the Fibonacci series.

1, 1, ___, ___, 5, ___, 13, 21, ___, ___

II. What is the fifteenth number in the Fibonacci series?

____

III. Draw a picture using only Fibonacci numbers (5 balloons, 13 apples)

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Unit 44

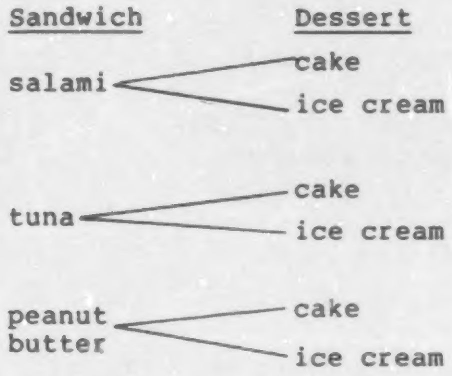
7.14.03 TREE DIAGRAMS
(ongoing)

PERFORMANCE OBJECTIVES

The student will be able to...

7.14.03.01 draw tree diagrams to represent the outcomes of problems;

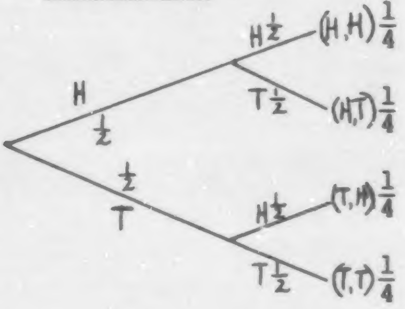
e.g.,


Sandwich
Dessert
salami
cake
ice cream
tuna
cake
ice cream
peanut butter
cake
ice cream

7.14.03.02 determine outcomes, events and probability when tossing one or more coins.

e.g.,

Two Tosses


H
1/2
1/2
T
H 1/2
T 1/2
H 1/2
T 1/2
(H,H) 1/4
(H,T) 1/4
(T,H) 1/4
(T,T) 1/4

P (2 tails) = $\frac{1}{4}$

P (2 heads) = $\frac{1}{4}$

7.14.03

VOCABULARY

TERMS

combinations
chance
probability
scale of probability

GETTING STARTED

Write the following lists on the chalkboard.

Girls		Boys	
2 dressy skirts	black maroon	2 pairs of dress pants	black navy
2 party blouses	white pink	2 dress shirts	white striped
2 pairs of shoes	pumps sandals	2 ties	red yellow

Say, "You are going to a friend's birthday party. How many possible combinations of outfits can you make from these choices?" Write the students' results on the chalkboard. Ask a student to count the number of combinations. Show the students the following way of organizing their work.

	Pants	Ties	Shirts
1)	Black (Bl)	Red (R)	White (W)
2)	Bl	R	Striped (S)
3)	Bl	Yellow (Y)	W
4)	Bl	Y	S
5)	Navy (N)	R	W
6)	N	R	S
7)	N	Y	W
8)	N	Y	S

Ask the students to organize the girls' outfits in a similar way. Ask several students to write their lists on the chalkboard and compare the results.

GETTING STARTED cont'd

Say, "The whole theory of probability began when scientists were asked by gamblers to answer questions about the fall of dice and other games of chance. Out of the research of such scientists as Pascal and Fermat came the laws of chance - Lady Luck unveiled.

These scientists picked quite a simple method, one which is still in use today."

Write the following on the chalkboard.

Scale of Probability		
Probability that water will flow downhill	Certain	1.00
Probability of sum of 13 on 2 dice	Impossible	0

Say, "When an event cannot possibly take place, the mathematician says, 'The chance is zero.' Thus the probability of getting a sum of 13 in a toss of two ordinary dice is 0, for such a sum is not possible. At the opposite end when an event must happen, such an occurrence is labeled 1."

Ask the students to describe some events that are impossible, e.g., running the mile in 1 minute, the sky turning green. Ask the students to list events that are certain, e.g., rain falling down, death, summer following spring.

Repeat this procedure using other events.

7.14.03

7.14.03.01 draw tree diagrams to represent the outcomes of problems

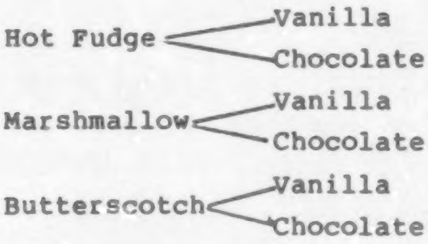
STRATEGY 1: Write this problem on the chalkboard.

> A menu offers ice cream sundaes with a choice of 2 kinds of ice cream and 3 different toppings. If each sundae consists of 1 flavor of ice cream with a topping, how many different sundaes are possible?

Allow the class about five minutes to find a solution. Write several responses on the chalkboard. Ask the class to count the number of possible combinations (6). Say that you are going to show them a yet simpler way of arriving at the number of possible sundae combinations.

Draw the following "tree diagram" on the chalkboard.

e.g.,



Say, "I have sketched what is known as a 'tree diagram', which is just another method of counting the different combinations."

Ask the class to draw tree diagrams to illustrate the following problem.

> A baseball team has 5 pitchers and 3 catchers. How many different combinations of pitchers and catchers can be made? (15)

Ask two or three students to draw their solutions on the chalkboard.

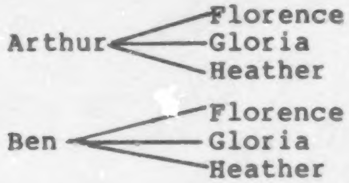
7.14.03

STRATEGY 2: Write the following on the chalkboard.

Boys	Girls
Arthur	Florence
Ben	Gloria
	Heather

Ask the students to list all possible pairs from the boys and girls. Say, "You must use one boy's name and one girl's name for each pair you list. Try to list your pairs in an organized way. How many pairs did you find?" Ask some students to write their lists on the chalkboard. Ask the students how many pairs they found. (6) Write the following on the chalkboard.

e.g.,



Ask the class if they can see the relationship between the diagram on the chalkboard and the list they have just completed. Say that the diagram on the chalkboard illustrates the tree diagram technique and is helpful in arrangement problems.

7.14.03.02 **determine outcomes, events and probability when tossing one or more coins**

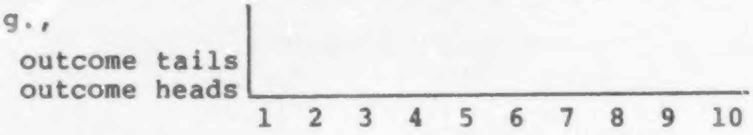
STRATEGY 1: Prepare one penny for each student. Toss a penny and catch it in your hand. Ask how many different ways it can land. (heads up or tails up) Say, "If a coin falls heads up, is it more likely to be tails up on the next toss? Let's toss some coins and find out."

7.14.03

STRATEGY 1: cont'd

Draw the following chart on the chalkboard.

e.g.,



Distribute one penny to each student. Ask the class to toss their coins ten times and after each toss to tabulate the results. Ask two or three students to record their tosses on the chalkboard. Repeat this procedure ten more times. Say, "If a coin is tossed 100 times, about how many times would you expect the coin to come up heads? (The most likely answer will be 50.) In other words, most of you would expect the outcome to be heads on half of the tosses. Although in the actual trials heads and tails were not equal outcomes, it seems reasonable to assume that each of these two events is equally likely to happen. Based on this assumption, we can say that the probability of heads is one out of two or $\frac{1}{2}$ and that the probability of tails is $\frac{1}{2}$." Write the following on the chalkboard.

e.g.,

$$P(H) = \frac{1}{2}, \quad P(T) = \frac{1}{2} \text{ where}$$

H → heads
T → tails
P → probability

Say, "What are the possible outcomes when you toss a penny and a nickel?" List the responses on the chalkboard.

e.g.,

	Penny	Nickel
Both are heads	H	H
Both are tails	T	T
head and tail	H	T
tail and head	T	H

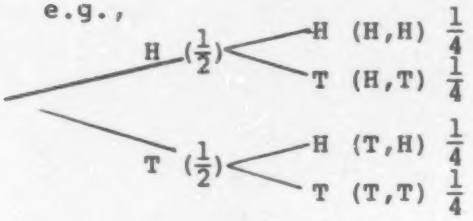
Say, "How many possible results are there when tossing two coins? (4) What are the chances of

STRATEGY 1: cont'd

getting all heads? (1 chance in 4 or $\frac{1}{4}$) of getting all tails? ($\frac{1}{4}$) ...of getting a combination of 1 head and 1 tail?" (2 chances out of 4 or $\frac{2}{4}$).

Draw a tree diagram on the chalkboard.

e.g.,



Say that each branch of the tree is labled $\frac{1}{2}$ because the probability of getting either heads or tails from each toss is $\frac{1}{2}$.

Repeat this procedure for three coins.

STRATEGY 2: Prepare enough coins for each pair of students. Ask the students to copy the following chart from the chalkboard as they are going to play "Two Heads are Better than One".

e.g.,

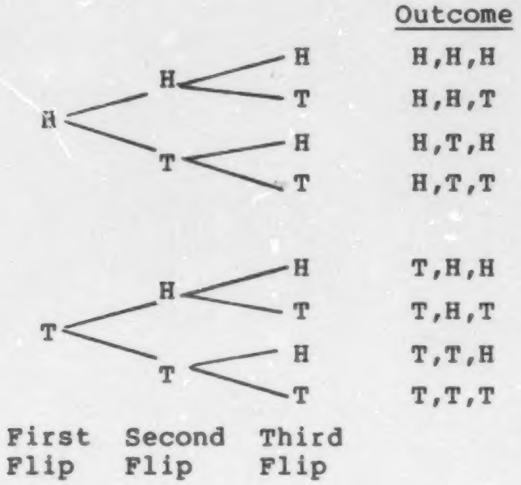
	1st Flip	2nd Flip	3rd Flip	Score
Round 1: Player 1				
Player 2				

Say, "Player 1 flips the coin three times recording each toss on the chart as either H (heads) or T (tails). Player 2 repeats this procedure. The one who gets 2 heads in 3 flips is the winner. If both players get 2 heads in 3 flips, no one wins the round. The winner scores 1 point." Ask the students to play four rounds.

At the end of the game, ask several of the playing pairs of students how many times their tally showed two heads in three flips. Record this information on the chalkboard.

STRATEGY 2: cont'd

Sketch a tree diagram on the chalkboard.
e.g.,


Outcome
H
H
T
H
T
H
T
H,H,H
H,H,T
H,T,H
H,T,T
T
H
T
H
T
H
T
T,H,H
T,H,T
T,T,H
T,T,T
First Flip
Second Flip
Third Flip

Say, "How many possible outcomes result from three flips of a coin? (8) How many times are three heads listed as a probability? (1) The probability of achieving three heads with three flips of a coin is one possibility out of eight possible outcomes or $\frac{1}{8}$. What other outcome has a probability of $\frac{1}{8}$? (3 tails) What is the probability of getting two heads with three flips? ($\frac{3}{8}$) What is the probability of getting more than one tail in three flips of a coin?" ($\frac{4}{8}$ or $\frac{1}{2}$)

P.O. 7.14.03.01

NAME:______________________________CLASS:________DATE:________

Use tree diagrams to solve the following problems.

1) How many 2-letter words can be formed using one of the vowels i or a, followed by one of the consonants n, t or s? Write all the words.

2) Menu A: soda, milk cocoa

Menu B: Muffin, pie

How many pairs of snacks can be formed by choosing one item from each menu. List all pairs.

3)

9AM - 10:30AM	10:30 - 12:00
Ceramics	Swimming
Arts & Crafts	Hiking
Weaving	Basketball
	Yoga
	Folk Dancing

To your left is the morning activity chart at camp Minniematha. Name all the possible activities available. How many pairs can be formed?

P.O. 7.14.03.02

NAME: ________________________ CLASS: ________ DATE: ________

A penny and a dime are tossed. Use the letters H and T for heads and tails.

1. List all the outcomes.

2. What is P (T,T)?

3. What is P (H,H)?

4. What is the probability that one coin will come up heads and the other tails?

420

7.14.04 PERMUTATIONS AND COMBINATIONS

(ongoing)

PERFORMANCE OBJECTIVES

The student will be able to...

7.14.04.01 define and make permutations;

e.g.,

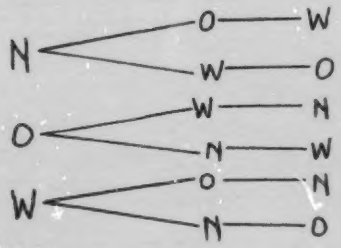
How many different permutations (arrangements) of the letters in the word now are possible?

now
nwo
onw
own
won
wno
(answer: 6)

7.14.04.02 use the counting principle to determine the number of permutations that can be made with a set of objects by formula;

e.g.,

To fill in the 3 blanks



$3 \times 2 \times 1 = 6$

7.14.04.03 define and make combinations.

e.g.,

How many committees of two students can be made from six students (Mary, Joe, Bill, Fran, Karen, Mark)?

1. Mary - Joe
2. Mary - Bill
3. Mary - Fran
4. Mary - Karen
5. Mary - Mark
6. Joe - Bill
7. Joe - Fran
8. Joe - Karen
9. Joe - Mark
10. Bill - Bran
11. Bill - Karen
12. Bill - Mark
13. Fran - Karen
14. Fran - Mark
15. Karen - Mark

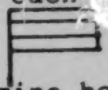
7.14.04

VOCABULARY

TERMS

permutation
arrangement

GETTING STARTED

Prepare and distribute to each student three different crayons and a flag outline as shown. 

Ask the students to determine how many ways a three stripe flag can be colored using a different color for each stripe. Ask the students to color their flags. Ask a few students to list their flag's colors on the chalkboard.

Repeat this procedure with four-stripe flags.

Say, "Suppose I gave a homework assignment such as, 'Choose any three of exercises 1, 2, 3, 4 or 5.' How many different choices can be made?"

Write on the chalkboard: I can choose to do 1, 2, 3 or 1, 2, 4.

Ask the students to continue listing choices as you write them on the chalkboard.

7.14.04

7.14.04.01 **define and make permutations**

STRATEGY 1: Say, "Make as many arrangements as you can of the letters in the word 'can'." Ask a few students to write their lists on the chalkboard. Say that each of the possible arrangements (CAN, CNA, ANC, ACN, NAC, NCA) is called a permutation.

Ask three students to come to the front of the classroom. Ask them to line up in as many different ways as they can. Keep a record of the permutations on the chalkboard. Ask how many permutations have been listed.

Repeat this procedure using other examples.

STRATEGY 2: Ask the class to consider the following example.

> A track coach has four students on her track team: Arlene, Bob, Cathy and Dan. The coach must decide in what order she will race the students so that they can win. How many arrangements are possible?

Ask the students for their estimates first, then make a list of the possible arrangements. Ask several students to place their lists on the chalkboard. Say that we call each arrangement a permutation. Say that there are six possible permutations once a beginning runner has been chosen.

7.14.04.02 **use the counting principle to determine the number of permutations that can be made with a set of objects by formula**

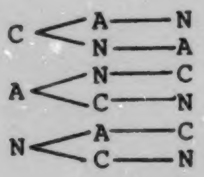
STRATEGY 1: Say, "Find the number of permutations that can be made from the letters of the word 'can'. (6) Another method of counting these permutations is to

STRATEGY 1: cont'd

construct a tree diagram."

Sketch the following on the chalkboard.

e.g.,



Say, "How many choices are there for the first letter of each permutation? (3) Having chosen the first letter, how many letters remain for my second choice? (2) How many letters remain for my third choice? (1) Here is a formula for an even quicker way of solving this permutation problem."

e.g.,

Number of permutations = P

P =	3	x	2	x	1	= 6
	choice for 1st letter		choice for 2nd letter		choice for 3rd letter	

Say, "In any permutation, we start with all possible choices and then continue."

Repeat this procedure using other examples.

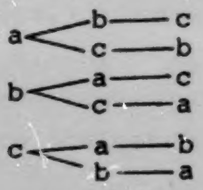
STRATEGY 2: Say, "For the letters a and b, how many permutations are possible? (2) What are they? (ab and ba) Given 3 elements such as a, b and c, list the permutations possible." Go around the room and ask several students who have made orderly lists to write them on the chalkboard.

e.g.,

abc bac cab
acb bca cba

Draw this tree diagram on the chalkboard.

e.g.,



STRATEGY 2: cont'd

Ask the following questions.

1) How many ways can the first letter be chosen? (3)
2) Having chosen the first letter, how many choices remain for the 2nd letter? (2)
3) In how many ways can the third place be filled? (1)

Say, "Any one of the three letters can be chosen for the first, and either of the two remaining letters can be chosen second. This leaves one letter to be placed in the third position. The number of permutations can be deduced in this way, 3 x 2 x 1 = 6. This method of counting permutations is called the Basic Counting Principle."

Repeat this procedure using other examples.

7.14.04.03 define and make combinations

STRATEGY 1: Ask four students to come to the front of the room. Ask how many four member committees can be formed. (1) Ask how many three member committees can be formed from the four students. Ask several students to write their lists at the chalkboard. If the students select two committees with the same people, e.g., Ben, John, Sue and Sue, Ben, John, say that these committees are considered as one. Say that combinations are different selections of members from a set without regard to their order.

Repeat this procedure for two member committees.

STRATEGY 2: Present this problem to the class.

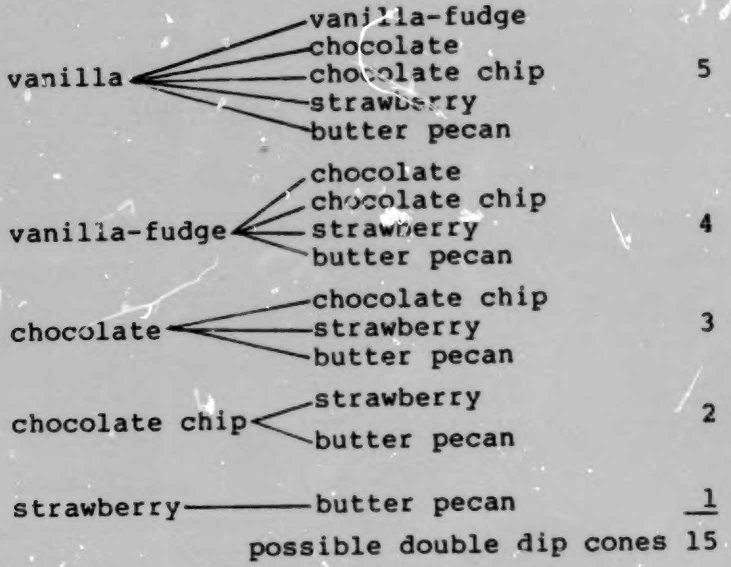
How many different double dip cones can be made from six different flavors: vanilla, vanilla-fudge, chocolate, chocolate chip, strawberry and butter pecan?

7.14.04

STRATEGY 2: cont'd

Suggest that the students use lists or tree diagrams. Ask several students to put their solutions on the chalkboard.

e.g.,


vanilla
vanilla-fudge
chocolate
chocolate chip
strawberry
butter pecan
5
vanilla-fudge
chocolate
chocolate chip
strawberry
butter pecan
4
chocolate
chocolate chip
strawberry
butter pecan
3
chocolate chip
strawberry
butter pecan
2
strawberry
butter pecan
1
possible double dip cones 15

Some students may come up with 30 double dip cones. Ask the class what happened to produce this many cones. (The same flavors were counted twice, e.g., vanilla - chocolate, chocolate - vanilla.) Say, "When you are not interested in the order by which a number of things are selected, but are interested only in the number of different items from a particular group, you are interested in combinations. Those of you who arrived at 30 different double dip combinations were actually making permutations, which in this case, turn out to be double the amount of combinations." (15)

P.O. 7.14.04.01-02

NAME: ______________________ CLASS: __________ DATE: __________

Show each permutation.

1. Six students sit in a row of six seats. In how many ways can these students be arranged in the six seats? (Use the Basic Counting Principle.)

2. How many ways can you arrange the letters in the word shape?

3. In a lottery, you must choose a four-digit number and match the winning number. If you can only use the digits 1,2,3,4, how many possible combinations are there?

7.12.04 FREQUENCY TABLES

(2-3 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

7.12.04.01A obtain the mode from the frequency table; e.g.,

Size 12 jeans is the mode because it has the largest frequency.

7.12.04.02A recognize how changes in data affect the mean, median and mode. e.g.,

	Salaries	
	Old Data	New Data
Bill	$10,000	$10,000
Joe	$12,000	$27,000
Frank	$43,000	$43,000
Susan	$26,000	$26,000

How will the mean, median and mode change?

7.12.04

VOCABULARY

TERMS

statistics
frequency
median
mode
mean
tally mark
data

GETTING STARTED

Say, "What is your favorite ice cream flavor?" Write the answer on the chalkboard, e.g., chocolate. Say, "Is chocolate anyone else's favorite flavor?"

Set up a frequency table like this one and record the answers. e.g.,

flavor	tally	number
chocolate	~~////~~	5

Ask another student for his/her favorite flavor, e.g., vanilla. Repeat the procedure. Your chart might look like this one. e.g.,

flavor	tally	number
chocolate	~~////~~	5
vanilla	~~////~~ /	6
chocolate chip	////	4
coffee	////	4
rocky road	~~////~~ //	7

Say, "This is called a frequency table. It enables us to see at a glance all of the data. We can see the most popular flavor, the least popular flavor, and those whose numbers are the same."

Repeat the procedure using various data.

Ask each student to name the floor that he/she lives on. Write each number on the chalkboard. Say, "We have all of the information, but let's organize it better."

GETTING STARTED cont'd

Write a frequency chart on the chalkboard.
e.g.,

Floor	Tally	Number

Ask a student to read each floor beginning with the smallest that was given. As he/she reads the number, record it on the frequency table and use tally marks to count the number of students who live on each floor.

Say, "When we have five tally marks, we show it by a diagonal slash over four tally marks, e.g., (~~////~~). This is called a frequency table. It helps us to organize our data."

Repeat this procedure using various data.

7.12.04

7.12.04.01 obtain the mode from the frequency table

STRATEGY 1: Say, "What is your favorite color?" Record the data in a frequency table.

e.g.,

color	tally	number
blue	~~////~~ //	7
yellow	~~////~~ ~~////~~ //	12
red	~~////~~ /	6
purple	~~////~~ ///	8

Say, "What is the most popular color? (yellow) We give a name to the most popular or most frequent item in our data. This is called the mode."

Repeat this procedure using various data.

STRATEGY 2: Place a set of test scores on the chalkboard.

e.g.,

80, 95, 70, 80, 85, 65, 90, 75, 60, 90, 100, 65,
85, 75, 80, 90, 95, 85, 65, 80, 70, 90, 70, 80,
85, 80, 60, 75, 80, 85, 95, 80, 70, 85

Ask the students to put these scores into a frequency table.

e.g.,

score	tally	number
60	//	2
65	///	3
70	////	4
75	///	3
80	~~////~~ ///	8
85	~~////~~ /	6
90	////	4
95	///	3
100	/	1

Ask a student which score was obtained most frequently. (80) Say, "We have a name for the

STRATEGY 2: cont'd most frequent item in our data. It is called the mode."

Repeat this procedure giving the students various data to put into frequency tables.

7.12.04.03 **recognize how changes in data affect the mean, median and mode**

STRATEGY 1: Ask ten students to take their pulse for one minute. Record the results on the chalkboard.

e.g.,

73, 75, 74, 73, 73, 76, 73, 75, 76, 76

Ask the class to put this into a frequency table and find the mode, mean and median.

e.g.,

pulse	tally	number
73	////	4
74	/	1
75	//	2
76	///	3

mode - 73
mean - 74.4
median - 74.5

Ask five of the same students to jog in place and take their pulse. Record their pulses on the chalkboard, e.g., 96, 92, 94, 92, 94. Ask the students to add this to the original pulse rates in the frequency table and find the mode, mean and median of the new table.

e.g.,

pulse	tally	number
73	///	3
74	/	1
75	/	1
92	//	2
94	//	2
96	/	1

7.12.04

STRATEGY 1: cont'd

Say, "What changes took place in the mean, mode and median. Why?"

Repeat this procedure by changing the data.

e.g.,

1) Add numbers to the total.
2) Lower the pulse rates.
3) Raise the pulse rates.

STRATEGY 2:

Prepare and distribute data to organize into a frequency table.

e.g., Favorite Sneakers

Sneaker	Tally	Number
Nike	~~////~~ ////	9
Adidas	~~////~~ ~~////~~ //	12
Pony	////	4
Keds	//	2

Ask the students to find the mode, mean and median. Now change the data by taking out the Adidas. Ask how this changes the mean, median and mode. Take out the Keds and do the same. Add a fifth sneaker and ask the students to find the changes. Discuss these changes, e.g., when we take out the Adidas which was the mode, the new mode becomes Nike (the second most frequent). Continue discussing the changes.

Repeat this procedure using various data.

P.O. 7.12.04.01

NAME: ______________________ CLASS: __________ DATE: __________

Make a frequency table for each set of data.

Example:

1. Ages of 2nd grade students:

6, 8, 7, 6, 6, 6½ 7, 7,

6, 6, 6½, 7, 7, 6, 6½

Age	Tally	Number
6	~~////~~ /	6
6½	///	3
7	~~////~~	5
8	/	1

2. Math test scores:

85, 80, 75, 85, 90, 100, 75, 75

75, 80 80, 85, 85, 60, 70, 75

3. Cars sold in one week:

22, 24, 23, 19, 31, 24, 22

4. Shoe size of basketball players:

10, 10½, 12, 12½, 11, 11, 11½

12, 12½, 13, 11, 12½, 13, 12½

435

P.O. 7.12.04.01

NAME: ________________________ CLASS: __________ DATE: ________

Organize the data into frequency tables and find the mode.

Example:

48, 49, 52, 55, 70, 64, 52, 56, 63, 52, 56, 63

Data	Tally	Number
48	/	1
49	/	1
52	///	3
55	/	1
56	//	2
63	//	2
64	/	1
70	/	1

Mode 52

1. 36, 31, 33, 36, 28, 31, 29, 31, 36, 32, 35, 36

2. 12, 16, 15, 17, 14, 16, 16

3. 11, 15, 10, 19, 10, 15, 11, 12, 11, 19, 14, 11, 15, 12, 11

4. 123, 125, 125, 123, 124, 128 123, 122, 123, 129, 123, 121, 122, 124

5. $7.22, $7.24, $7.22, $7.23, $7.22, $7.23, $7.24, $7.20, $7.24

P.O. 7.12.04. 02

NAME:______________________________________CLASS:_________DATE:_______

What is the change in each set of statistics?

Example: On a pre-test in spelling 10 students get:

75, 80, 75, 90, 60, 60, 75, 80, 85, 75

On the re-test the same students received:

90, 100, 95, 95, 85, 90, 95, 100, 95, 90

What is the change in the mode, mean, median?

Pre-Test			Re-Test		
60	11	2	85	1	1
75	1111	4	90	111	3
80	11	2	95	1111	4
85	1	1	100	11	2
90	1	1			
Mean	75.5 .75 75		Mean	93.5 95 95	

1. A new member joined the track team, make 2 frequency tables one of the weights old team & one of the weights new team. What is the difference in the mean, mode, median.

Old team		New team	
120	110	120	150
130	120	130	120
135	130	125	130

2. A store is taking inventory in its stock-when it adds new items, how does that change the mean, mode, & median.

Old stock		New stock	
3.50	3.75	3.50	3.75
3.75	3.50	3.75	3.50
4.50	4.00	4.50	4.00
3.50	3.50	4.00	3.50
2.00	4.50	3.50	4.50

7.12.05 COMPOUND PROBABILITY

(2-3 lessons)

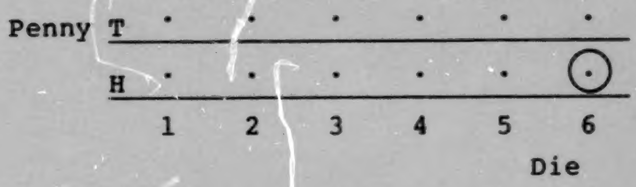
PERFORMANCE OBJECTIVES

The student will be able to...

7.12.05.01B determine the probability of two independent events occurring together by using a sample space;

e.g.,

What is the probability of a head and a six occurring at the same time?

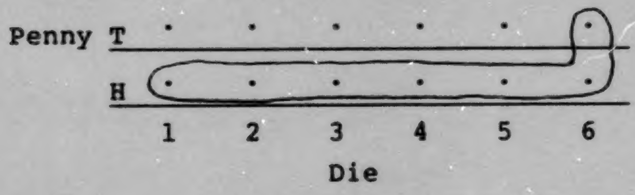


$P(\text{head and } 6) = \frac{1}{12}$

7.12.05.02B determine the probability of either of two outcomes occurring by using a sample space.

e.g.,

What is the probability of a head or a six occurring at the same time?



$P(\text{head or } 6) = \frac{7}{12}$

7.12.05

VOCABULARY

TERMS

predict
chance
compound

GETTING STARTED

Write the following statements on the chalkboard.

1) Horses are smarter than pigs.
2) If you touch a frog you'll get warts.
3) Lightning never strikes twice in the same place.
4) Breaking a mirror is bad luck.
5) If you toss a coin three times and it always comes up heads, the fourth time it will come up tails.

Ask the students if they believe these statements to be true. Ask the students if they can think of some other things people believe in that are false, e.g., knock on wood for luck, bad things will happen if you walk under a ladder. Say, "Most of us believe in things that are not true. Why?" Elicit answers such as the following.

1) Someone who we trust told us.
2) We read it somewhere.
3) Sometimes we jump to conclusions.
4) The law of averages.

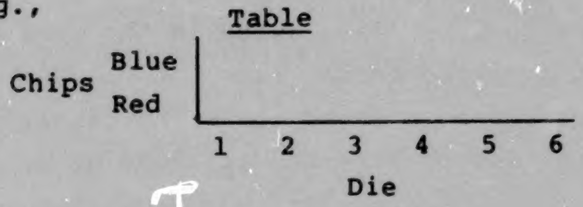
Ask the students if they've ever heard anyone say, "Things couldn't get worse." or "The only way I can go is up." Say, "In a coin toss game, if the last four tosses were heads, the law of averages says the next toss will come up tails." Ask a student if this is true. Say, "If it rains five days in a row, according to the law of averages, the sixth day will be sunny. Is this true?" Ask the students if they can think of some other things that people believe that are not true.

7.12.05

7.12.05.01 **determine the probability of two independent events occurring together by using a sample space**

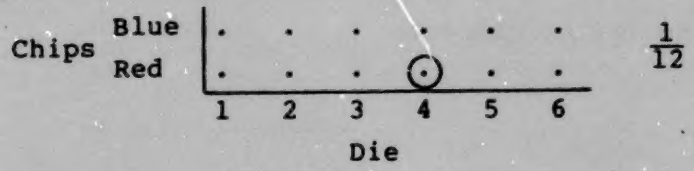
STRATEGY 1: Prepare a blue and red chip and a die. Put the blue chip and the red chip into a bag and roll the die. Say, "What are my chances of rolling a four with the die and choosing a red disc at the same time? Make a table of all the possibilities. How many possibilities are on the die? (6) How many possibilities do you have with the chips?" (2)

e.g.,



Ask a student to demonstrate, using the table, the possibility of rolling a 4 on the die. (one out of six or $\frac{1}{6}$) Ask a student what the possibilities are of choosing a red disc. (one out of two or $\frac{1}{2}$) What are the possibilities of drawing a red disc and rolling a four on the die?

e.g.,

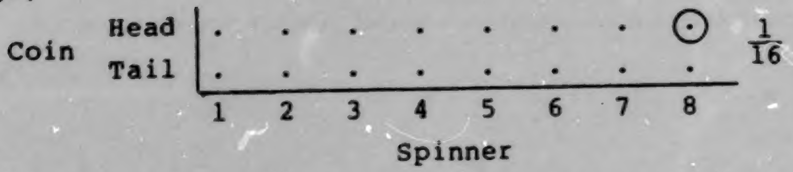


Repeat this procedure with other problems.

STRATEGY 2: Prepare and display a coin and a spinner divided into eight equal parts to the class. Ask the students what the outcomes are when tossing a coin. (heads or tails) What are the outcomes of spinning a number. (1,2,3,4,5,6,7,8) Ask a student what the chances are of tossing a coin and getting heads. ($\frac{1}{2}$) What is the probability of tossing a head and spinning an 8?

STRATEGY 2: cont'd

e.g.,

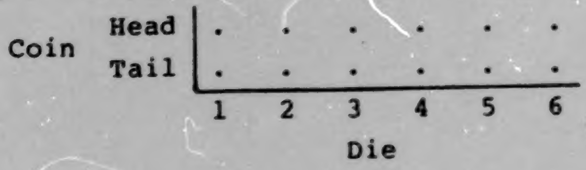


Repeat this procedure with other probabilities.

7.12.05.02 **determine the probability of either of two outcomes occurring by using a sample space**

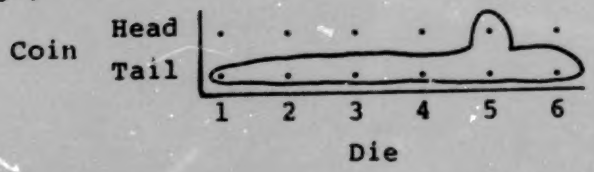
STRATEGY 1: Prepare and display a coin and a die. Write the following chart on the chalkboard.

e.g.,



Say, "What are the total number of outcomes shown in the chart? (12) If we were to toss a coin and roll a die together, what events could happen if we wanted a head to come up or a three?" (head and 1, head and 2, head and 3, head and 4, head and 5, head and 6) List these on the chalkboard. Say, "If we show these on the chart, they would look like this."

e.g.,



Say, "How many possibilities are circled? (7) What is the probability of getting a head or a five?" $(\frac{7}{12})$

Repeat this procedure using other combinations.

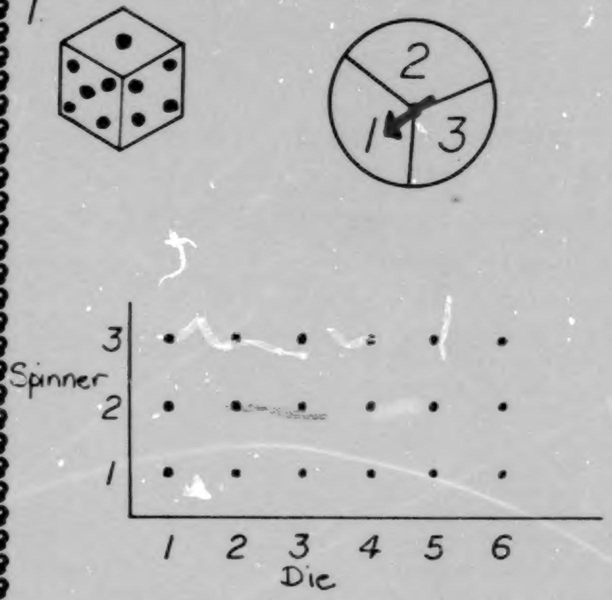
STRATEGY 2: Adapt Strategy 1 used in P.O. 7.12.05.01 for determining the probability of either of two outcomes by using a chart.

P.O. 7.12.05.01.-.02

NAME: ______________________ CLASS: __________ DATE: ________

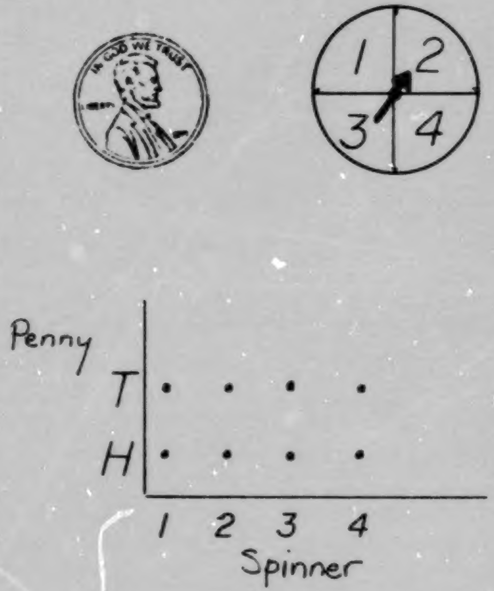
Use the sample space to determine the probability of each experiment.

1.


2
1
3
Spinner
3
2
1
1 2 3 4 5 6
Die

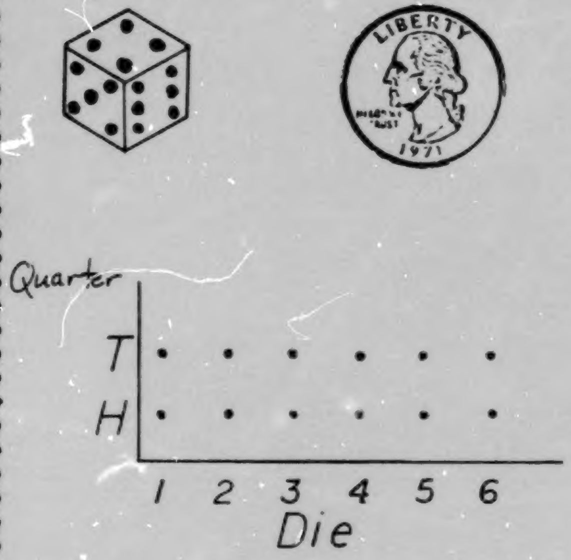
P (2 on the spinner and 5 on the die) =

2.


IN GOD WE TRUST
1
2
3
4
Penny
T
H
1 2 3 4
Spinner

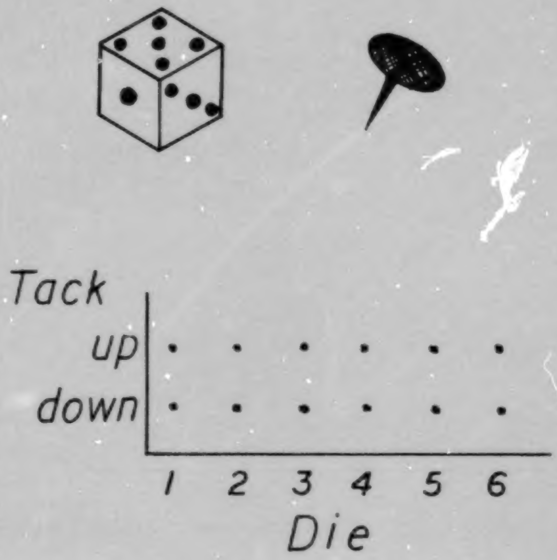
P (tail and a 3) =

3.


LIBERTY
1971
Quarter
T
H
1 2 3 4 5 6
Die

P (head or 4) =

4.


Tack
up
down
1 2 3 4 5 6
Die

P(up or 2) =

Unit 48

7.13.04 PATTERNS

(1-2 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

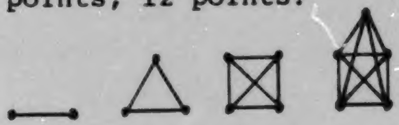
7.13.04.01A identify and extend a numerical pattern;
e.g.,

a) 6, 12, 18, 24 ...
What is the rule?
What is the next number?

b) 1, 2, 4, 7, 1, 16 ...
What is the rule?
What is the next number?

7.13.04.02A use patterns to solve problems.
e.g.,

How may lines are needed to join 9 points, 12 points?



7.13.04

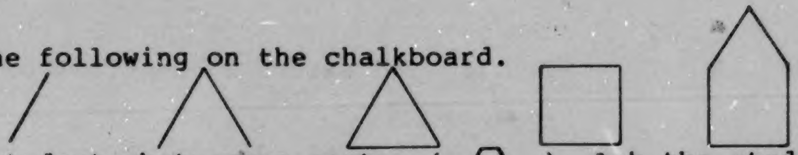
VOCABULARY

TERMS

pattern
progression
series

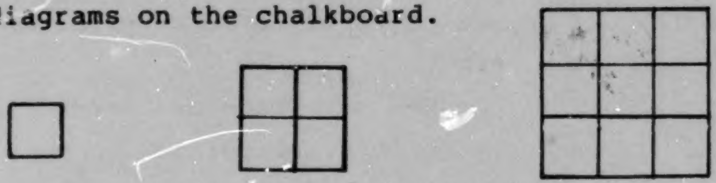
GETTING STARTED

Draw the following on the chalkboard.



Ask a student what comes next. (⬡) Ask the student how he/she came to that conclusion. (First, there was one line segment, then, two line segments, then, three line segments, then, four and five. Therefore, six line segments must be in the next figure.) Say that this is called a pattern. Ask the class if anyone can name another pattern that he/she knows. (Times tables are patterns.) Say that all patterns have rules, and the trick to working with patterns is to discover the rule. Write the following diagrams on the chalkboard.

e.g.,



Say, "Who can guess the rule? How many little boxes will be in the next diagram?" (1x1, 2x2, 3x3... and the number of little boxes in the next diagram will be 16.)

Ask the class to find the next three pictures in this pattern now that they know the rule.

Draw a calendar on the chalkboard.

e.g.,

S	M	T	W	Th	F	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

GETTING STARTED cont'd

Ask the students if they can find any interesting patterns on this calendar. (If we go across horizontally in any row, all the numbers increase by one; if we go down any column, all the numbers increase by seven; if we go down diagonally from right to left, all the numbers increase by six; if we go down diagonally from left to right, all the numbers increase by eight.)

Ask the class to discover different sequences. Draw an array with nine numbers on one line and repeat the above procedure.

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7.13.04

7.13.04.01 **identify and extend a numerical pattern**

STRATEGY 1: Draw Pascal's triangle on the chalkboard as follows.

e.g.,

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

Ask a student if he/she could tell what the next row of numbers will be. (1,7,21,35,35,21,7,1) Ask another student if he/she could identify the rule for finding each succeeding row. (Add two numbers in a line to find the number that goes beneath both of them.)

e.g.,

1 , 7 , 21 , 35 , 35 , 21 , 7 , 1
1 8 28 56 70 56 28 8 1

Ask the class if they know of any other sequences that follow a rule. (Street addresses, room numbers, tickets, and even multiplication tables all follow rules.)

Place the following sequence on the chalkboard.

e.g.,

5, 8, 11, 14, ☐

Ask a student to figure out the rule to find the next number in the sequence. (5+3 = 8, 8+3 = 11, and 11+3 = 14. Therefore, the rule is "add three to each number to get the next number.")

Repeat this procedure using other sequences.

STRATEGY 2: Say, "I am going to write on the chalkboard several numbers that are in a pattern. You have to guess the rule for the pattern and then figure out the

STRATEGY 2: cont'd

next number in the series."

Write the following on the chalkboard.

e.g.,
1, 6, 11, 16 ...

Ask a student to figure out the rule. (5 + 1 = 6, and 6 + 5 = 11, and 11 + 5 = 16, and 16 + 5 = 21)

Write this next series on the chalkboard and ask another student to guess the rule and the next number.

e.g.,
1, 2, 4, 8, ☐

(We have to multiply each number by 2 to get the next number; the missing number is 16.)

Continue writing patterns on the chalkboard, each time eliciting the rule involved and the next number in the series.

7.13.04.02 **use patterns to solve problems**

STRATEGY 1: Write the following problem on the chalkboard.

> Mr. Smolinski is having a problem. He is having a party with six people and he is trying to figure out how many different ways the six people can be arranged. Can you figure out how many different combinations exist for six people to sit at a table?

Say, "This problem deals with combinations of six different things. If we try to do it without planning, it could take forever. Let us try to figure out a pattern to make it easier to do."

Suggest to the class that first we should work with 1 person, then 2 people, then 3 and so forth. Say, "How many different ways can 1 person sit? (1) If 2 people, A and B, are at a table, how many ways can they be arranged? (AB or BA) How many ways can 3

STRATEGY 1: cont'd

people be arranged?" (ABC, ACB, BAC, BCA, CAB, CBA) Repeat this procedure until you reach six people. Say that we can see a pattern of multiplying the product of the previous combination with the new number of people to get the number of combinations.

e.g.,

People		Combinations
1	1 x 1 = 1	1
2	1 x 2 = 2	2
3	2 x 3 = 6	6
4	6 x 4 = 24	24
5	24 x 5 = 120	120
6	120 x 6 = 720	720

Repeat this procedure using other examples.

STRATEGY 2: Prepare and distribute six square pieces of construction paper 1" x 1" to each student. Say, "Using two squares, how many ways do we have to arrange them?" (one) Repeat this procedure for three squares. (or). Continue for four and five squares. Say, "Let's make a chart of our experiment."

e.g.,

No. of Squares	Ways of Arranging Them
2	1
3	2
4	5
5	12

Ask the students if they can see a pattern. (We can find the next number of arrangements by multiplying the previous ways of arrangement by two and adding the previous number, e.g., for four we multiply, (2x2)+1 = 5, for five, (5x2)+2 = 12, and the next number in the series, (12x2)+5 = 29.)

Repeat this procedure using other problems.

P.O. 7.13.04.01-.02

NAME: ______________________ CLASS: ________ DATE: ________

For each of the following patterns, name the rule and find the next number.

1. 1, 2, 4, 8 ______

2. 2, 2, 3, 3, 4, 4, ______

3. 4, 14, 24, 34, ______

4. 7, 10, 13, 16, ______

5. 2, 5, 9, 14, 20, ______

6. 10, 11, 9, 10, 8, 9, ______

7. 12, 15, 18, 21, ______

8. 90, 88, 84, 78, 70, ______

9. 96, 48, 24, 12, 6, ______

10. 1, 4, 9, 16, 25 ______

7.14.05 POSITIVE AND NEGATIVE INTEGERS

(ongoing)

PERFORMANCE OBJECTIVES

The student will be able to...

7.14.05.01 identify an integer and use positive and negative integers in real life situations; e.g.,

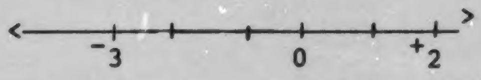
4 is an integer

6.2 is not an integer

Real Life Situations

Above and below sea level,
Above and below zero temperature,
Withdrawals and deposits,
Winning and losing

7.14.05.02 locate positive and negative integer on a number line; e.g.,



7.14.05.03 compare and order positive and negative integers. e.g.,

$^{+}4 > {}^{-}3$

$^{-}17 < {}^{-}10$

$^{-}8 = {}^{-}8$

$^{+}3, {}^{-}6, {}^{-}8, {}^{+}4$

in increasing order are:
$^{-}8, {}^{-}6, {}^{+}3, {}^{+}4$

7.14.05

VOCABULARY

TERMS

positive
negative
order

GETTING STARTED

Write the following on the chalkboard.

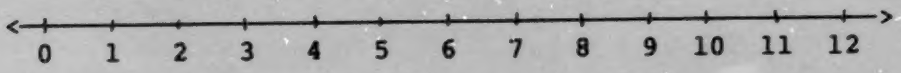
e.g.,

$$A = \{1,2,3,4 \ldots\}$$
$$B = \{0,1,2,3 \ldots\}$$

Ask the students to identify the sets of numbers in A (the set of counting numbers) and in B (the set of whole numbers).

Draw a number line on the chalkboard.

e.g.,



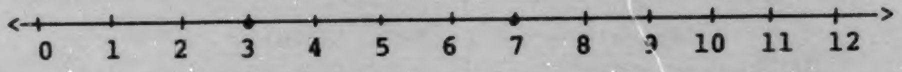
Write the following statements on the chalkboard.

e.g.,

$3 < 7, 2 > 0, 8 < 10$

Ask the students to locate each pair of numbers on the number line. For each pair graph the numbers.

e.g.,



$3 < 7$

Say that in each pair of numbers, the larger one is always on the right on the number line.

Repeat this procedure with other pairs of numbers.

7.14.05

7.14.05.01 **identify an integer and use positive and negative integers in real life situations**

STRATEGY 1: Write the following numbers on the chalkboard.

e.g.,

I	II
7	5.2
12	13.16
83	96.05
156	356.283
5,000,000	5.2 million

Ask what the difference is between the numbers listed in column I and the numbers listed in column II. (Numbers in column I are whole numbers and the numbers in column II are decimals.) Say, "Whole numbers are integers, while decimals are not." If you are climbing a hill and you climbed 30 feet, then fell and rolled back 40 feet, how far would you have climbed?" (A loss of ten feet or $^{-}10$) Say, "Negative and positive whole numbers are integers, also."

Repeat this procedure using other situations.

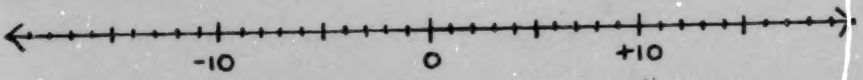
STRATEGY 2: Say, "In one play in a football game, the Giants gained 8 yards. During the same play the umpire gave the Giants a 15 yard penalty. How can we represent each situation by using a number? ($^{+}8$ for gain of eight yards, $^{-}15$ for a penalty of fifteen yards) We call signed whole numbers integers."

Repeat this procedure with other questions, e.g., weight loss, temperature changes...

7.14.05

7.14.05.02 **locate positive and negative integers on a number line**

STRATEGY 1: Construct a number line on the chalkboard (or construct a number line from strips of oak tag for permanent use and mount it at the top ledge of the chalkboard) and number through $^{+}10$ and $^{-}10$.
e.g.,



Ask the students to construct their own number line. Ask what numbers are in the set of integers. (The set of integers includes zero and all of the whole numbers above and below zero, e.g., $^{-}3$, $^{-}2$, $^{-}1$, 0, $^{+}1$, $^{+}2$, $^{+}3$.

Ask a student to come to the chalkboard and locate $^{+}5$. Repeat this procedure with other integers, e.g., $^{-}7$, $^{-}1$, $^{+}3$, $^{+}8$.

STRATEGY 2: Construct a number line on the chalkboard from $^{+}10$ to $^{-}10$. Label four points: A, B, C and D. Ask the students to name the numbers at points A, B, C and D. Repeat this procedure with other points on the number line.

7.14.05.03 **compare and order positive and negative integers**

STRATEGY 1: Ask the students to construct a number line or to use a prepared number line. Write the following pair of integers on the chalkboard, $^{-}3$ $^{+}1$. Ask the students to locate these points on the number

STRATEGY 1: cont'd

line and determine which sign they would put in the circle (>, <, or =). Repeat this procedure with other pairs of integers. Say that when comparing two integers on a number line, the number to the right is always the larger integer.

Write a few integers on the chalkboard, e.g., 0, $^-2$, $^+1$, $^-4$. Ask the students to locate and graph each number on the number line. Ask a student to rewrite the integers, in order, from the smallest to the largest. Repeat this procedure with other integers.

STRATEGY 2:

Ask the students to pretend to use an elevator, which goes from the eighth floor to three floors below the ground level, to answer the following questions. Explain that they are starting on the eighth floor and are going to the third floor. Say, "Which floor is higher, $^+8$ or $^+3$?" Repeat this procedure by going up or down in the elevator.

P.O. 7.14.05.01

NAME: ______________________________ CLASS: ________ DATE: _______

Write positive or negative next to each situation.

1. The temperature is 60° above zero. ____________________

2. You owe 10 dollars. ____________________

3. You deposit 100 dollars in your bank account. ____________________

4. You dive 3 meters below the surface of the water. ____________________

5. The football team gained twelve yards. ____________________

6. There are 5 new students in class. ____________________

7. You live on the 8th floor. ____________________

8. You lost 12 lbs. ____________________

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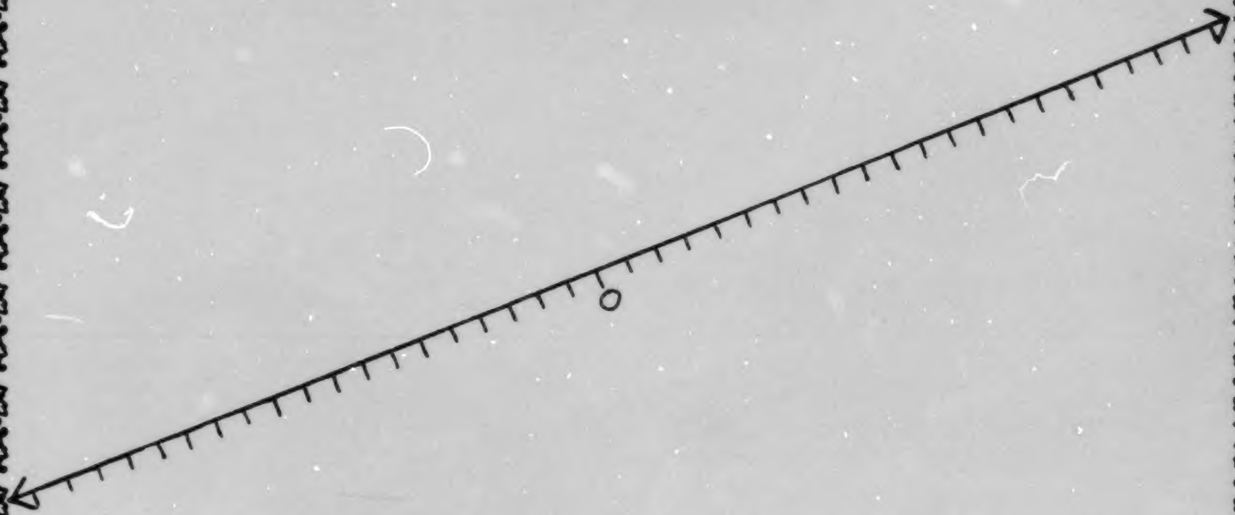
P.O. 7.14.05.02

NAME: ________________________________ CLASS: ________ DATE: ________

Locate and label each point on the number line.



① $^{-}16$ ⑥ $^{+}20$

② $^{+}3$ ⑦ $^{-}2$

③ $^{-}8$ ⑧ $^{+}5$

④ $^{-}19$ ⑨ 0

⑤ $^{+}7$ ⑩ $^{+}12$

456

P.O. 7.14.05.03

NAME:______________________________ CLASS:__________ DATE:________

I. Use < or > to make the following sentences true.
II. List in order from the smallest to the largest integer.

I.

1. 0 ☐ 4
2. 4 ☐ 2
3. -2 ☐ -4
4. -5 ☐ +5
5. -9 ☐ 0
6. -18 ☐ -23
7. +50 ☐ 22
8. -12 ☐ -18
9. 100 ☐ +100
10. -7 ☐ -2 ☐ 2

II.

1. 5, -3, -7, 0, 2, 1 ______________________________
2. 12, 3, 0, -8. -11, 4 ______________________________
3. 9, 7, 1, 12, -9 ______________________________
4. 0, -3, 8, 4, -16, 11 ______________________________

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Unit 50

7.08.01 SUBTRACTION OF SIGNED NUMBERS

(2-3 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

7.08.01.01A subtract positive and negative integers using the inverse operation of addition; e.g.,

To solve $(^{+}3) - (^{-}2) = \square$,

we use $(^{-}2) + \square = {}^{+}3$.

$(^{-}2) + (^{+}5) = {}^{+}3$

7.08.01.02A subtract a negative integer from a negative integer using the inverse operation of addition.

e.g.,

To solve $(^{-}7) - (^{-}2) = \square$,

we use $(^{-}2) + \square = {}^{-}7$.

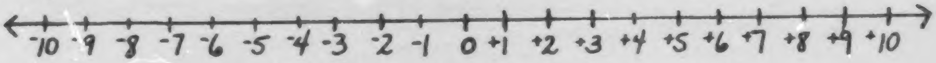
$(^{-}2) + (^{-}5) = {}^{-}7$

GETTING STARTED

Write the following problems on the chalkboard.

$$\begin{array}{llllllll} 1) & \begin{array}{r} {}^{+}5 \\ +\ \underline{{}^{+}3} \end{array} & 2) & \begin{array}{r} {}^{-}5 \\ +\ \underline{{}^{-}3} \end{array} & 3) & \begin{array}{r} {}^{+}5 \\ +\ \underline{{}^{-}3} \end{array} & 4) & \begin{array}{r} {}^{-}5 \\ +\ \underline{{}^{+}3} \end{array} \end{array}$$

Ask the students to find the sum in each problem. Ask a student to use the number line to define the rules for addition of integers. e.g.,



Repeat this procedure with other problems using a horizontal format.

1) ${}^{+}6 + {}^{+}2 =$

2) ${}^{-}6 + {}^{-}2 =$

3) ${}^{-}6 + {}^{+}2 =$

4) ${}^{+}6 + {}^{-}2 =$

Write the following problems on the chalkboard.
e.g.,

$$\begin{array}{cccc} \begin{array}{r} 5 \\ \underline{-2} \end{array} & \begin{array}{r} 8 \\ \underline{-5} \end{array} & \begin{array}{r} 9 \\ \underline{-7} \end{array} & \begin{array}{r} 12 \\ \underline{-4} \end{array} \end{array}$$

Ask the students to find each difference. Ask the students to check each problem. Repeat this procedure with other problems using a horizontal format, e.g., $6 - 5 = \boxed{}$.

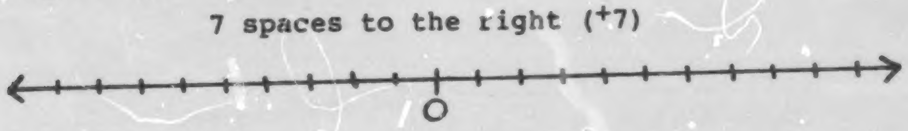
7.08.01

7.08.01.01 subtract positive and negative integers using the inverse operation of addition

STRATEGY 1: Write $^{+}4 - (^{-}3) = \square$ on the chalkboard. Say, "Subtraction is finding the missing addend. We can do this problem if we think about an addition problem, e.g., $^{-}3 + \square = {}^{+}4$.

Plot this on a number line.

e.g.,



$^{-}3 + \boxed{^{+}7} = {}^{+}4$ or $^{+}4 - (^{-}3) = \boxed{^{+}7}$

Repeat this procedure using other subtraction problems.

STRATEGY 2: Write the following problem on the chalkboard, $^{+}7 - (^{+}5)$. Say, "What do you add to positive 5 to get positive 7?" Write this on the chalkboard, $^{+}5 + \square = {}^{+}7$. Say that $^{+}7 - {}^{+}5 = \square$, and $^{+}5 + \square = {}^{+}7$ are inverse operations and that we can solve a subtraction problem by writing a related addition problem. Ask the students to solve $^{+}5 - (^{-}7)$ using the same procedure, e.g., $^{+}5 - (^{-}7) = \square$ and $^{-}7 + \square = {}^{+}5$. ($^{+}12$)

Repeat this procedure with other subtraction problems using both horizontal and vertical formats.

7.08.01

7.08.01.02 **subtract a negative integer from a negative integer using the inverse operation of addition**

STRATEGY 1: Write the following problems on the chalkboard.

e.g.,

$$
\begin{aligned}
{}^{-}3 - ({}^{+}1) &= {}^{-}4 \\
{}^{-}3 - (0) &= {}^{-}3 \\
{}^{-}3 - ({}^{-}1) &= {}^{-}2 \\
{}^{-}3 - ({}^{-}2) &= {}^{-}1 \\
{}^{-}3 - ({}^{-}3) &= 0 \\
{}^{-}3 - ({}^{-}4) &= {}^{+}1 \\
{}^{-}3 - ({}^{-}5) &= {}^{+}2 \\
{}^{-}3 - \square &= \square
\end{aligned}
$$

Ask the students if they can continue the pattern. (${}^{-}3 - ({}^{-}6) = {}^{+}3$) Repeat this a few times.

Say, "Do we have to write a chart of problems each time we subtract a negative integer from a negative integer?" (We can use the number line to solve problems.) Ask a student if there is any other way to solve a subtraction problem. (Each subtraction problem can be written as a related addition problem, e.g., ${}^{-}3 - ({}^{-}1) = \square$, can be written as ${}^{-}1 + \square = {}^{-}3$.) Ask the students to write the other problems as related addition problems and solve.

Repeat this procedure with other problems.

STRATEGY 2: Adapt Strategy 2 used in P.O. 7.08.01.01 for subtracting a negative integer from a negative integer using the inverse operation of addition.

P.O. 7.08.01.01

NAME: ______________________ CLASS: ________ DATE: ________

Write the inverse of each problem and solve.

1. $^{+}18 - {}^{-}4 =$

2. $^{-}12 - {}^{+}7 =$

3. $^{+}2 - {}^{-}8 =$

4. $^{-}4 - {}^{+}11 =$

5. $^{-}8 - {}^{+}14 =$

6. $^{+}9 - {}^{-}14 =$

7. $^{+}19 - {}^{-}19 =$

8. $^{-}24 - {}^{+}24 =$

P.O. 7.08.01.02

NAME:____________________ CLASS:________ DATE:_______

Write each problem as a related additon problem and solve.
Find the message.

1. $^{+}7 - {}^{+}4 =$

$^{+}4 + \square = {}^{+}7$

$^{-}3$ D

$^{+}3$ [✓ T]

2. $^{-}10 - {}^{-}12 =$

$^{+}2$ T

$^{-}2$ O

3. $^{-}14 - {}^{-}7 =$

$^{-}7$ K

$^{+}7$ I

4. $^{-}9 - {}^{-}11 =$

$^{-}2$ T

$^{+}2$ E

5. $^{-}7 - {}^{-}4 =$

$^{-}3$ F

$^{+}3$ O

6. $^{-}11 - {}^{-}5 =$

$^{-}16$ V

$^{-}6$ I

7. $^{-}4 - {}^{-}8 =$

$^{+}4$ V

$^{-}12$ E

8. $^{-}7 - {}^{-}13 =$

$^{+}6$ E

$^{-}20$ R =

I ___ ___ ___ ___ ___ ___ ___

1 2 3 4 5 6 7 8

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7.08.02 MULTIPLICATION OF SIGNED WHOLE NUMBERS

(1-2 lessons)

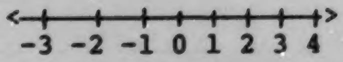
PERFORMANCE OBJECTIVES

The student will be able to...

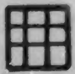
7.08.02.01A	multiply a positive whole number by another positive whole number; e.g., $^{+}3 \times {}^{+}2 = {}^{+}6$
7.08.02.02A	multiply a negative whole number by a positive whole number; e.g., $^{-}3 \times {}^{+}2 = {}^{-}6$
7.08.02.03A	multiply a negative whole number by another negative whole number. e.g., $^{-}3 \times {}^{-}2 = {}^{+}6$

7.08.02

VOCABULARY

TERMS	**SYMBOLS**
positive	+
negative	-
number line	

GETTING STARTED

Prepare and distribute a grid similar to this  to each student, or write it on the chalkboard and ask the students to copy it in their notebooks. Ask the students to fill in four integers in four squares as demonstrated . Ask the students to fill in the other squares so that the sum of the first two numbers is placed in the third square both horizontally and vertically.

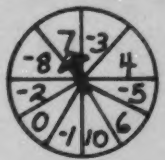
e.g.,

4	-7	-3
-3	2	-1
1	-5	-4

Repeat this procedure using other grids and ask the students to fill in positive and negative integers.

Prepare a spinner with ten positive and negative integers on it.

e.g.,



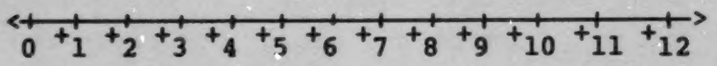
Play "Integer Addition." Divide the students into two teams, A and B. One player from Team A spins the spinner and a player from Team B writes the integer on which the pointer lands on the chalkboard. The procedure is repeated so that two integers are on the chalkboard, e.g., $^{-}3$ $^{+}10$. The player from Team B adds the integers and writes the answer on the chalkboard, e.g., $^{-}3 + {}^{+}10 = {}^{+}7$. If he/she is correct, Team B gets a point. Play continues with the teams alternating between spinning and adding.

7.08.02

7.08.02.01 **multiply a positive whole number by another positive whole number**

STRATEGY 1: Draw a number line on the chalkboard from $^{-}20$ to $^{+}20$. Write $^{+}3 \times 0 =$ on the chalkboard. Ask a student to solve the example, e.g., $^{+}3 \times 0 = 0$. Then write $^{+}3 \times {}^{+}1 = \square$. Ask another student to write the answer, e.g., $^{+}3 \times {}^{+}1 = {}^{+}3$. Repeat for $^{+}3 \times {}^{+}2 = \square$ and $^{+}3 \times {}^{+}3 = \square$. On the number line, draw the progression.

e.g.,



Say, "What would you expect $^{+}3 \times {}^{+}4$ to equal? ($^{+}12$) Why?" (Multiples of three moving to the right on the number line.) Repeat this procedure for $0 \times {}^{+}3$, $^{+}1 \times {}^{+}3$, $^{+}2 \times {}^{+}3$, $^{+}3 \times {}^{+}3$, $^{+}3 \times {}^{+}4$. Say, "Do the products change? Why not? (commutative property of multiplication) Let's look at our factors. What is the same about them? (positive integers) What is the same about the products? (positive integers) What rule do you notice about a product if the factors are both positive?" (The product is a positive integer.) Write the rule on the chalkboard.

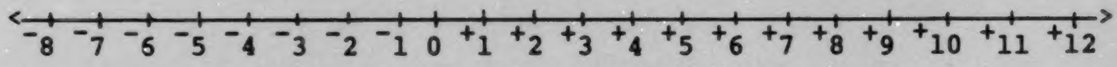
STRATEGY 2: Say, "How can we represent a bank deposit of \$15? ($^{+}15$) Suppose we deposit \$15 for five weeks in a row. What algorithm will show this? ($^{+}15 \times {}^{+}5$) How much money will you have deposited in this five week period?" (\$75) Repeat using other deposits for several amounts of weeks, e.g., \$10 for three weeks, \$8 for nine weeks. Ask the students what the sign of the product is each time. (positive) Say, "When multiplying a positive integer by another positive integer, the product is always positive."

7.08.02

7.08.02.02 **multiply a negative whole number by a positive whole number**

STRATEGY 1: Draw a number line on the chalkboard.

e.g.,



Ask the students to compute the following examples to see if they can find a pattern.

e.g.,

$^{+}3 \times {}^{+}4 = (12)$
$^{+}3 \times {}^{+}3 = (9)$
$^{+}3 \times {}^{+}2 = (6)$
$^{+}3 \times {}^{+}1 = (3)$
$^{+}3 \times 0 = (0)$

Make a loop on the number line to represent the answers. Say, "What do you think 3 x $^{-}1$ would equal if it follows this pattern? ($^{-}3$) What would 3 x $^{-}2$ equal?"

Reinforce that multiplication is commutative by having the students reverse the same numbers.

e.g.,

$^{+}4 \times {}^{+}3 = \square$
$^{+}3 \times {}^{+}3 = \square$
$^{+}2 \times {}^{+}3 = \square$
$^{+}1 \times {}^{+}3 = \square$
$^{+}1 \times {}^{+}3 = \square$
$^{-}1 \times {}^{+}3 = \square$
$^{-}2 \times {}^{+}3 = \square$

Say, "What would the next answer be using the number line?" ($^{-}3 \times {}^{+}3 = {}^{-}9$) Ask the students what type of numbers $^{-}1$ and $^{+}3$ are. (negative and positive numbers) Say, "What kind of number do we get when we multiply a positive and a negative factor? (negative) Can you think of a rule for multiplying positive and negative factors?" (When multiplying a positive by a negative number, the product is negative.) Write the rule on the chalkboard.

STRATEGY 2: Write the following on the chalkboard.

A girl loses two pounds a week for three weeks.
How much lighter is she?

Ask a student to represent the loss of two pounds on the chalkboard. ($^{-}2$) Ask the students to write an example which represents the problem. ($^{-}2$ x $^{+}3$ =) Say, "How many pounds did the girl lose? (6) How can we show that she is six pounds lighter?" ($^{-}6$)

Repeat this procedure using an example about withdrawing $5 from a bank account each week for four weeks. Ask the students to represent the loss of money to the account by using an example involving positive and negative factors and products. Ask the students to state a rule about multiplying a positive factor by a negative factor. Write the rule on the chalkboard, e.g., When multiplying a positive factor by a negative factor, the product is always negative.

7.08.02.03 **multiply a negative whole number by another negative whole number**

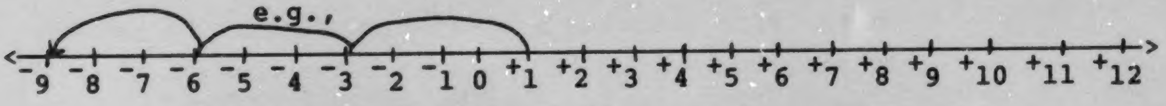
STRATEGY 1: Draw a number line on the chalkboard. Ask the students to compute the following examples.

e.g.,

$^{-}3 \text{ x } ^{+}3 = \square$
$^{-}3 \text{ x } ^{+}2 = \square$
$^{-}3 \text{ x } ^{+}1 = \square$
$^{-}3 \text{ x } 0 = \square$

Make a loop on the number line to represent the answers.

e.g.,



Say, "What is the pattern? (The product is 3 spaces to the left each time.) Using the number

STRATEGY 1: cont'd

line, what would $^-3 \times {}^-1$ be?" $({}^-3 \times {}^-1 = {}^+3)$ Repeat this procedure using $^-3 \times {}^-2$, $^-3 \times {}^-3$, ... Say, "What do we notice about the sign of the product when we multiply a negative number by a negative number?" (positive product)

Write the following on the chalkboard.

e.g.,

Complete the following sentences:

1) The product of a positive integer and a negative integer is a ____________ integer.
2) The product of a positive integer and a positive integer is a ____________ integer.
3) The product of a negative integer and a negative integer is a ____________ integer.

STRATEGY 2: Write the following on the chalkboard.

For each of the last two weeks, Mary has withdrawn $25 from the bank. How much more money did she have 2 weeks ago?

(She had $50 more than she has now or ${}^+\$50$.) Ask a student come up to the chalkboard and represent the sentence using integers.

e.g.,

$$\underset{\text{last 2 weeks}}{({}^-2)} \times \underset{\text{withdrawal}}{({}^-25)} = {}^+50$$

Say that a negative integer multiplied by a negative integer is a positive integer.

Repeat this procedure using other examples of banking or weight.

P.O. 7.08.02.01

NAME: ______________________ CLASS: __________ DATE: ________

Find the product of each example.

1) $^{+}3$ x $^{+}7$ = ____________

2) $^{+}9$ x $^{+}8$ = ____________

3) $^{+}13$ x $^{+}8$ = ____________

4) $^{+}26$ x $^{+}3$ = ____________

5) $^{+}37$ x $^{+}12$= ____________

6) $^{+}97$ x $^{+}21$= ____________

7) $^{+}10$ x $^{+}12$= ____________

8) $^{+}13$ x $^{+}14$= ____________

9) $^{+}7$ x $^{+}14$ = ____________

10) $^{+}6$ x $^{+}5$ = ____________

11) $^{+}4$ x $^{+}9$ = ____________

12) $^{+}3$ x $^{+}11$= ____________

13) $^{+}2$ x $^{+}9$ = ____________

14) $^{+}1$ x $^{+}2$ = ____________

15) $^{+}5$ x $^{+}16$= ____________

16) $^{+}8$ x $^{+}40$= ____________

17) $^{+}53$ x$^{+}21$= ____________

18) $^{+}79$ x $^{+}3$= ____________

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P.O. 7.08.02.02.-03

NAME: ______________________ **CLASS:** ________ **DATE:** ________

Find each product.

1. ${}^{-}9$ x ${}^{-}12$ = ________
2. ${}^{-}13$ x ${}^{-}10$ = ________
3. ${}^{+}9$ x ${}^{-}8$ = ________
4. ${}^{-}16$ x ${}^{+}4$ = ________
5. ${}^{+}5$ x ${}^{+}4$ = ________
6. ${}^{-}6$ x ${}^{+}7$ = ________
7. ${}^{+}14$ x ${}^{+}12$ = ________
8. ${}^{+}7$ x ${}^{-}5$ = ________
9. ${}^{-}3$ x ${}^{-}3$ = ________
10. ${}^{-}2$ x ${}^{+}8$ = ________
11. ${}^{+}15$ x ${}^{+}3$ = ________
12. ${}^{-}18$ x ${}^{-}5$ = ________
13. ${}^{-}8$ x ${}^{-}3$ = ________
14. ${}^{+}4$ x ${}^{-}2$ = ________
15. ${}^{+}1$ x ${}^{+}3$ = ________
16. ${}^{-}10$ x ${}^{-}2$ = ________
17. ${}^{-}5$ x ${}^{-}3$ = ________
18. ${}^{-}2$ x ${}^{+}3$ = ________
19. ${}^{-}3$ x ${}^{-}4$ = ________
20. ${}^{+}7$ x ${}^{-}3$ = ________

7.08.03 DIVISION OF SIGNED WHOLE NUMBERS

(2-3 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

7.08.03.01B	compute the quotient with a positive dividend and a positive divisor; e.g., $^{+}36 \div {}^{+}6 = {}^{+}6$
7.08.03.02B	compute the quotient with a positive dividend and a negative divisor; e.g., $^{+}36 \div {}^{-}6 = {}^{-}6$
7.08.03.03B	compute the quotient with a negative dividend and a positive divisor; e.g., $^{-}36 \div {}^{+}6 = {}^{-}6$
7.08.03.04B	compute the quotient with a negative dividend and a negative divisor. e.g., $^{-}36 \div {}^{-}6 = {}^{+}6$

GETTING STARTED

Write the following on the chalkboard: always, never, present and absent. Ask a student to describe whether the words have a positive or negative value. (The words always and present have a positive value, while never and absent have a negative value.) Say, "We are going to combine two of these words and see what happens to the value of the total statement. If someone is always present is that positive or negative? (positive) If someone is always absent is that positive or negative? (negative) If someone is never absent is that negative or positive? (positive) If someone is never present is that negative or positive?" (negative) Draw the following chart.

e.g.,

	+ always	- never
+ present	+	-
- absent	-	+

Say, "This chart shows the outcomes when we multiply positive and negative numbers."

Place the following multiplication problems on the chalkboard.

1) $^+8 \times {}^+6$ 2) $\begin{array}{r} ^+7 \\ \underline{\times\ {}^-3} \end{array}$ 3) $^-8 \times {}^-3$ 4) $\begin{array}{r} ^+9 \\ \underline{\times\ {}^-6} \end{array}$

Ask the class to find the products using the rules of multiplication of integers. (When doing a multiplication problem with signed numbers, the product is positive when the factors have similar signs, and negative when the factors have different signs.) Write the following on the chalkboard.

pos x pos = pos (+ x + = +) neg x pos = neg (- x + = -)
pos x neg = neg (+ x - = -) neg x neg = pos (- x - = +)

Prepare oak tag index cards. Divide the class into two groups. Ask each group to make up flashcards with multiplication problems using integers, e.g., $\boxed{^-6 \times {}^+5.}$ Ask each group to write the answers on the back of the card. Ask a student from each group to flash one card at the opposite group. Say to the class that the winning team will receive a plus, and that's positive!

7.08.03

7.08.03.01 **compute the quotient with a positive dividend and a positive divisor**

STRATEGY 1: Ask the students to divide 8 by 2. (This division may be expressed as $\frac{8}{2}$ or 8÷2 and the quotient is 4.) Ask if it is possible to test the result. (8 ÷ 2 = 4 can be checked by multiplication since multiplication is the inverse (opposite) operation of division.) Say, "To divide 8 by 2 we find a number which, when multiplied by 2, gives 8, e.g., since 4 x 2 = 8 (quotient x divisor = dividend), the problem checks out.

Ask the students to solve, $^+8 \div {}^+4$. (We must find a number that when multiplied by $^+4$ gives $^+8$. The required number is $^+2$, e.g., $^+2 \times {}^+4 = {}^+8$. Therefore, $^+8 \div {}^+4 = {}^+2$.)

Repeat this procedure using other division problems with a positive dividend and a positive divisor.

STRATEGY 2: Ask the students to write a multiplication and division problem using each set of three numbers, e.g., (5,6,30), (7,6,42), (15,5,3), (9,81,9). Say, "What is the relationship between multiplication and division? (inverse operations) We can find a quotient using this relationship." Write the following on the chalkboard.

e.g.,

Multiplication	Division
$^+7 \times {}^+3 = {}^+21$	$^+21 \div {}^+3 = \square$
$^+9 \times {}^+6 = {}^+54$	$^+54 \div {}^+6 = \square$
$^+10 \times {}^+3 = {}^+30$	$^+30 \div {}^+3 = \square$

Ask several students to solve the division examples using the related multiplication example. Say, "What sign is in each quotient, when the dividend

STRATEGY 2: cont'd and divisor are both positive?" (positive) Ask the students to solve several division examples with positive dividends and divisors.

7.08.03.02 compute the quotient with a positive dividend and a negative divisor

STRATEGY 1: Ask the students to find the quotient of $^{+}8 \div {}^{-}2$. (We must find a number which when multiplied by $^{-}2$ will give $^{+}8$, and $^{-}4$ is that number. So, $^{+}8 \div {}^{-}2 = {}^{-}4$.) Ask the students to check the division by using the related multiplication fact, e.g., $^{-}4 \times {}^{-}2 = {}^{+}8$.

Repeat this procedure using similar division problems with a positive dividend and a negative divisor. Say that the quotient is always a negative number when the divisor and dividend are opposite signs.

STRATEGY 2: Write the following on the chalkboard.

If $^{-}3 \times {}^{-}6 = {}^{+}18$, what does $^{+}18 \div 6 = \boxed{?}$

Ask a student to write the answer on the chalkboard, e.g., $^{+}18 \div {}^{-}6 = {}^{-}3$. Repeat this procedure using the following sentences.

If $^{-}6 \times {}^{-}6 = {}^{+}36$, what does $^{+}36 \div {}^{-}6 = \boxed{?}$
If $^{-}8 \times {}^{-}3 = {}^{+}24$, what does $^{+}24 \div {}^{-}3 = \boxed{?}$

Say, "What rule is demonstrated by these examples concerning the sign of a quotient whose dividend is positive and whose divisor is negative? (The quotient will be negative.) Unlike signs in division yield a negative quotient." Write $(^{+}) \div (^{-}) = (^{-})$ on the chalkboard.

7.08.03

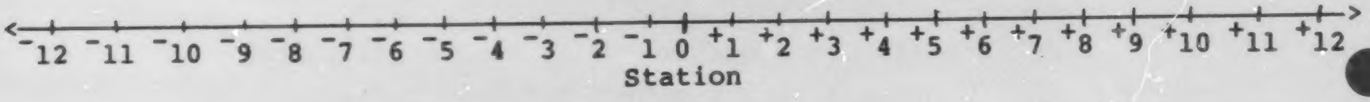
7.08.03.03 **compute the quotient with a negative dividend and a positive divisor**

STRATEGY 1: Ask the students to find the quotient of $^{-}8 \div {}^{+}2$. (We need to find a number which when multiplied by $^{+}2$ gives $^{-}8$. The number is $^{-}4$. So, $^{-}8 \div {}^{+}2 = {}^{-}4$.)

Ask the students to check the answer using the inverse operation of multiplication, e.g., $^{-}4 \times {}^{+}2 = {}^{-}8$.

Repeat this procedure using other examples. Say that the quotient of a negative dividend and a positive divisor is always a negative number.

STRATEGY 2: Draw the following "train track" on the chalkboard. e.g.,



Discuss the meaning of the "train track". Say, "The station is at zero and the positive direction is to the right, the negative direction is to the left. What would $^{+}3$ miles per hour mean? (go three to the right) ...$^{-}4$ per hour? (go four to the left) ...$^{+}5$ hour? (Five hours after the train passes the station.) ...$^{-}3$ hours?" (Three hours before the train passes the station.)

Write the following on the chalkboard.

> A train is traveling at six mph in the negative direction. Where will it be two hours after it leaves the station?

Ask a student to write a multiplication example to solve this problem and show it on the "track." ($^{-}6 \times {}^{+}2 = \square$) Ask another student to solve the problem. ($^{-}6 \times {}^{+}2 = 12$)

Write the following example on the chalkboard.

STRATEGY 2: cont'd

A train is standing at post $(^{-}12)$. It got to this post after two hours of traveling. At what rate did the train travel?

Ask a student to write a division example to solve this problem and show it on the "track." $(^{-}12 \div {}^{+}2 = {}^{-}6)$ Repeat this procedure using similar problems.

Say, "What is the sign of the quotient when we have a negative dividend and a positive divisor? (negative) In division, two unlike signs yield a negative quotient. $(^{+} \div {}^{-} = {}^{-})$

7.08.03.04 **compute the quotient with a negative dividend and a negative divisor**

STRATEGY 1: Ask the students to find the quotient of $^{-}8 \div {}^{-}2$. (We need to find a number which when multiplied by $^{-}2$, gives $^{-}8$. The number is $^{+}4$. So $^{-}8 \div {}^{-}2 = {}^{+}4$.) Ask the students to check using the inverse operation of multiplication, e.g., $^{+}4 \times {}^{-}2 = {}^{-}8$.

Repeat this procedure using other examples. Say that the quotient of a negative dividend and a negative divisor is always a positive number.

STRATEGY 2: Adapt Strategy 2 used in P.O. 7.08.03.02 for computing the quotient with a negative dividend and a negative divisor.

P.O. 7.08.03.01

NAME: ______________________ CLASS: __________ DATE: ________

Divide.

Example: THINK

$^{+}36 \div {}^{+}6 = \boxed{6}$ or $\boxed{6} \times {}^{+}6 = {}^{+}36$

This number is the product

of these factors

1. $\frac{^{+}16}{^{+}2} =$

2. $^{+}36 \div {}^{+}12 =$

3. $\frac{^{+}18}{^{+}6} =$

4. $^{+}6 \div {}^{+}6 =$

5. $^{+}80 \div {}^{+}8 =$

6. $\frac{^{+}64}{^{+}16} =$

7. $\frac{^{+}75}{^{+}3} =$

8. $+ 96 \div + 6 =$

P.O. 7.08.03.02-03

NAME: ____________________ CLASS: __________ DATE: __________

I. Write true or false.

II. Divide.

I. 1. $^{-}10 \div {}^{+}5 = {}^{+}2$ ________ 2. $^{-}21 \div {}^{+}7 = {}^{-}4$ ________

3. $^{+}25 \div {}^{-}5 = {}^{-}5$ ________ 4. $^{+}9 \div {}^{-}3 = {}^{-}3$ ________

5. $^{-}18 \div {}^{+}3 = {}^{+}6$ ________ 6. $^{-}42 \div {}^{+}7 = {}^{-}6$ ________

II. 7. $\frac{200}{-4} =$ 8. $^{+}4 \div {}^{-}4 =$

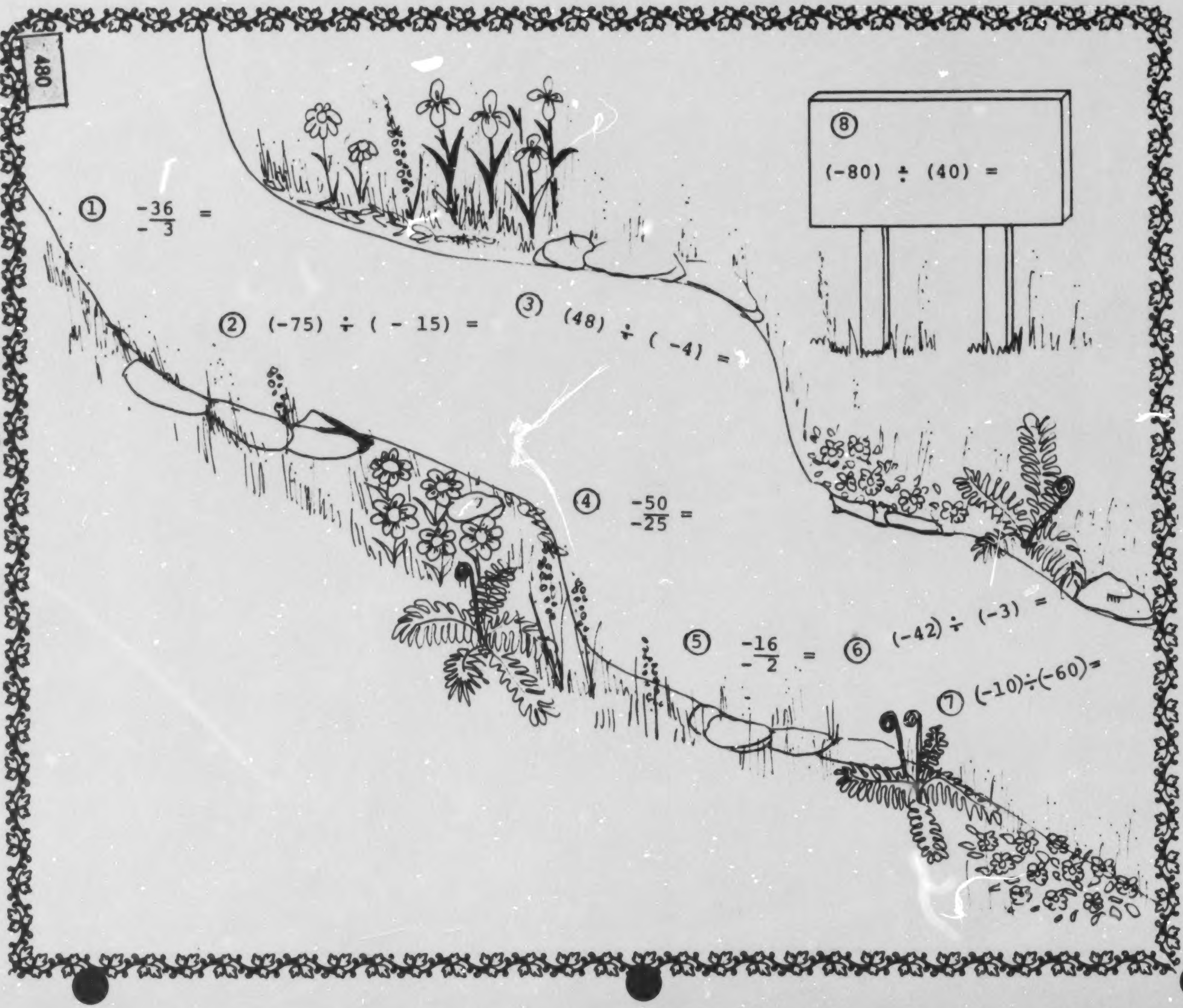
9. $\frac{-100}{^{+}25} =$ 10. $\frac{-55}{^{+}11} =$

11. $^{-}24 \div {}^{+}6 =$ 12. $\frac{^{+}45}{-9} =$

P.O. 7.08.03.04

NAME: ______________________ CLASS: __________ DATE: ________

Divide.



① $\frac{-36}{-3} =$

② $(-75) \div (-15) =$

③ $(48) \div (-4) =$

④ $\frac{-50}{-25} =$

⑤ $\frac{-16}{-2} =$

⑥ $(-42) \div (-3) =$

⑦ $(-10) \div (-60) =$

⑧ $(-80) \div (40) =$

480

Unit 53

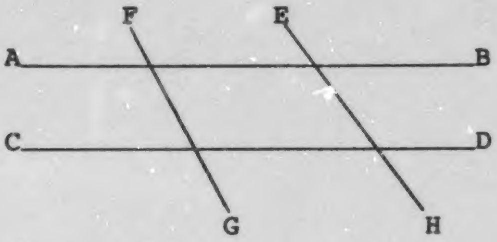
7.09.06 PARALLEL LINES AND THEIR PROPERTIES

(3 lessons)

PERFORMANCE OBJECTIVES

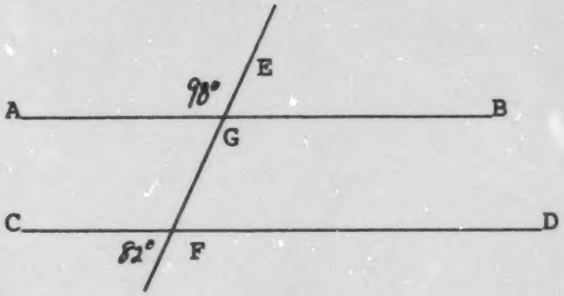
The student will be able to...

7.09.06.01A identify a pair of parallel lines; e.g.,



$\overline{AB}$ and $\overline{CD}$ are parallel.

7.09.06.02B determine pairs of alternate interior angles and corresponding angles formed by a transversal through parallel lines; e.g.,

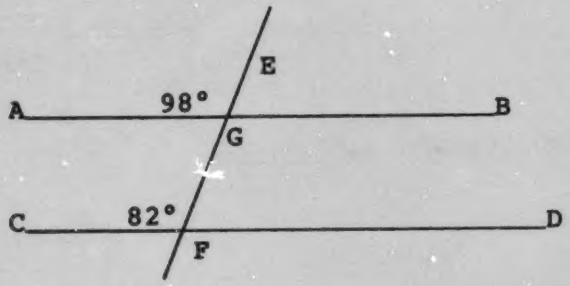


∠ BGF and ∠ CFG are alternate interior angles. They are equal.

∠ EGB and ∠ GFD are corresponding angles. They are equal.

7.09.06.03B apply the properties of parallel lines to numerical problems.

e.g.,



∠ BGF =

∠ BGF = 98° since it is the vertical angle of ∠ AGE.

7.09.06

VOCABULARY

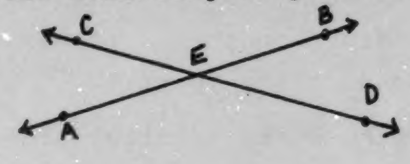
TERMS

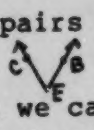
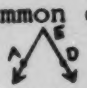
parallel lines
transversal

GETTING STARTED

Draw the following diagram on the chalkboard.

e.g.,



Ask the students to name and list pairs of rays having E as a common end point. (ray $\overrightarrow{EC}$ and $\overrightarrow{EB}$,  , and ray $\overrightarrow{EA}$ and $\overrightarrow{ED}$,  ...) Ask the students what we call two rays having a common end point. (angles) Ask the students to determine the type of each angle. (∡ CEB is obtuse, ∡ CEA is acute...) Ask the students to name the vertical and supplementary angles. (∡ CEB and ∡ AED are vertical angles, ∡ CEB and ∡ BED are supplementary angles...)

Repeat this procedure using other diagrams.

Prepare and distribute to each student a sheet with several different angles drawn on it and a protractor. Ask the students to measure the angles on their sheets using the protractor. Emphasize the correct method of measuring using a protractor.

7.09.06

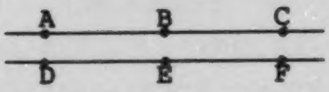
7.09.06.01 **identify a pair of parallel lines**

STRATEGY 1: Ask the students to define what parallel lines are. List the answers on the chalkboard.

Draw a pair of parallel lines on the chalkboard. Ask the students what they have seen in their daily lines that resembles parallel lines. Say, "Has anyone traveled on the subway? Do the train tracks look like the two lines drawn on the chalkboard? Are they always the same distance apart?"

Repeat this procedure using other parallel lines in the environment.

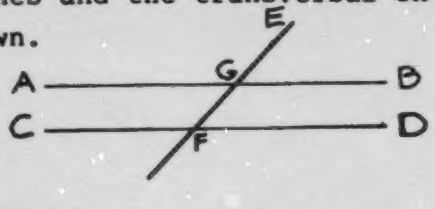
STRATEGY 2: Draw a pair of parallel lines on the chalkboard. e.g.,



Ask a student to measure the distances $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$. Say that parallel lines are always the same distance apart and never intersect one another. Ask the class if they have seen examples of parallel lines. Challenge the students to look around the classroom and find as many pairs of parallel lines as they can. Allow the students to work in teams and see which team can find the greatest number of pairs.

7.09.06.02 **determine pairs of alternate interior angles and corresponding angles formed by a transversal through parallel lines**

STRATEGY 1: Label parallel lines and the transversal on the chalkboard as shown. e.g.,



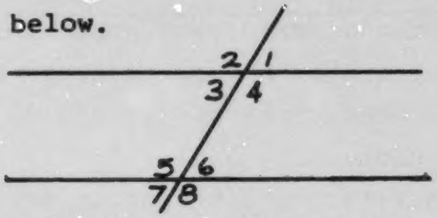
STRATEGY 1: cont'd

Ask a student to measure angles BGF and CFG at the chalkboard. Say that these two angles are formed when a transversal is drawn through two parallel lines, and that each angle is on opposite sides of the transversal and between the parallel lines. Say that these angles are called alternate interior angles and have the same measure. Ask the class to find another set of alternate interior angles, e.g., $\angle$ AGF and $\angle$ DFG.

STRATEGY 2:

Ask the students to draw any transversal through a pair of parallel lines in their notebooks. Ask them to measure the alternate interior angles with their protractors. Ask several students to come to the chalkboard and draw the figures they have drawn in their notebooks. Say that each transversal forms two sets of alternate interior angles. Number the angles in the original drawing as indicated below.

e.g.,



Write the following on the chalkboard and ask several students to justify why the statements are true.

1) $m\angle 1 = m\angle 3$ (vertical angles)
2) $m\angle 3 = m\angle 6$ (alternate interior angles)
3) Therefore, $m\angle 1 = m\angle 6$

Say that these angles, which are formed by a transversal cutting a pair of parallel lines and which lie on the same side as well as being above or below the parallel lines, are always equal to each other and are called corresponding angles.

Ask the class if they can find other pairs of

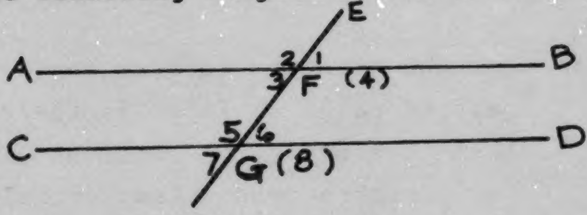
STRATEGY 2: cont'd corresponding angles in the diagram on the chalkboard. (∡2 and ∡5, ∡3 and ∡7, ∡4 and ∡8.)

Repeat this procedure using other diagrams.

7.09.06.03 **apply the properties of parallel lines to numerical problems**

STRATEGY 1: Draw the following diagram on the chalkboard.

e.g.,



Say, "What is the measure of ∡AFG? (80°) Why?" (Vertical angles are equal.) Point to another pair of vertical angles (∡2, ∡4). Say, "How can we determine the measure of these angles without using a protractor? (Each angle measures 100° because they are both supplementary to angle 1. The measures of ∡2 and ∡1 add up to 180°.) Name other supplementary angles." (∡2 and ∡3, ∡1 and ∡4, ∡4 and ∡3) Write the angle measures on the chalkboard.

e.g.,

m ∡1 = 80°
m ∡2 = 100°
m ∡3 = 80°
m ∡4 = 100°

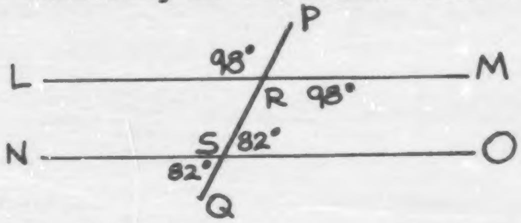
Using the properties of alternate interior and corresponding angles, find the measures of ∡5, ∡6, ∡7 and ∡8 in the diagram.

Repeat this procedure using other diagrams.

7.09.06

STRATEGY 2: Draw the following on the chalkboard.

e.g.,



Ask the students to name all alternate interior angles in the diagram and discuss their properties. Say, "How can we determine the measures of each angle without using a protractor?" (∡MRS = 98°, therefore, ∡RSN = 98°, ...) Repeat this procedure for corresponding angles. Fill in the measures of all angles in the diagram.

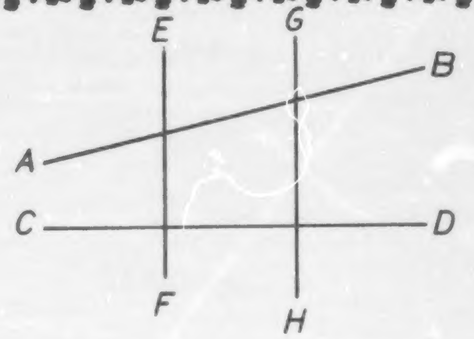
Repeat this procedure using other diagrams.

P.O. 7.09.06.01-03

NAME: ______________________________ CLASS: __________ DATE: ________

(1) Identify parallel lines.

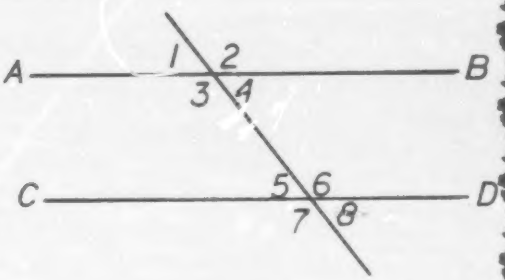
(2) Identify transversals.



(3) Identify all vertical angles.

(4) Identify all alternate interior angles.

(5) Identify all corresponding angles.



Refer to the above figure for the following examples:

(6) If m∡1 is 40°, what is m 4? ______

(7) If m ∡ 2 is 140°, what is m 3? __________

(8) Angles 3 and 4 are called __________________ angles.

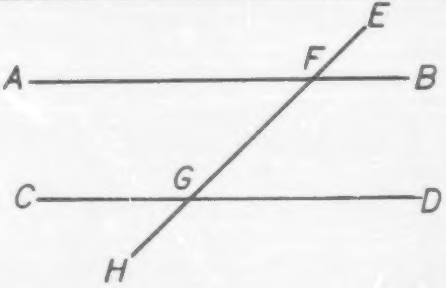
AB // CD
M∡FGD = 72°
Find the measure of these angles. Do not use a protractor.

(9) ∡ AFG

(10) ∡ EFB

(11) ∡ AFE

(12) ∡ BFG



Unit 54

7.09.07 ANGLES OF A POLYGON

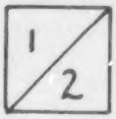
(4-5 lessons)

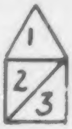
PERFORMANCE OBJECTIVES

The student will be able to...

7.09.07.01B find the sum of the angle measures of a polygon;

e.g.,





2 triangles = 180° + 180° = 360°

3 triangles = 180° + 180° + 180° = 540°

Generalize: The sum of the angle measures of any convex n-gon is (n-2)180° where n is the number of sides.

7.09.07.02B find the sum of the angles of any polygon in numerical problems;

e.g.,

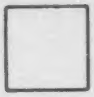
Find the measure of the fourth angle of a quadrilateral when the other three angles measure: 55°, 81°, 103°.

360° - (55 + 81 + 103)° =

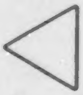
360° - 239° = 121°

7.09.07.03B identify a regular polygon (equilateral and equiangular);

e.g.,



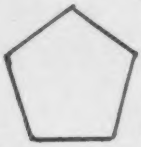
regular



not regular

7.09.07.04B find each interior angle of a regular polygon in numerical problems.

e.g.,



Each angle = 108°

7.09.07

VOCABULARY

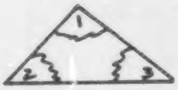
TERMS

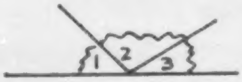
regular polygon
symetrical

GETTING STARTED

Cut out a large oak tag triangle and label the angles 1, 2 and 3. Say, "What is the sum of the angles of any triangle? (180°) How many degrees are there in an acute angle? (less than 90°) There are 90° in a right angle, there are more than 90° but less than 180° in an obtuse angle, and there are 180° in a straight angle. If a straight angle has 180° and a triangle also has 180°, is there a way to show that the angles of a triangle make a straight angle?" (We can show this by tearing off angles 1, 2 and 3 and putting them together to form a straight angle.)

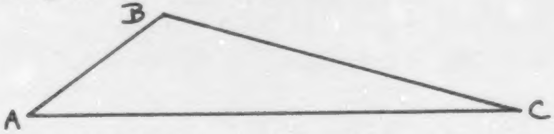
e.g.,





Draw a triangle on the chalkboard.

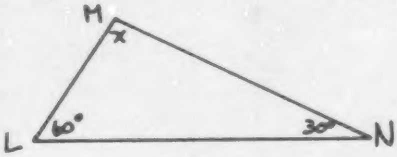
e.g.,



Ask a student to measure ∢ A, ∢ B and ∢ C. Say that the sum of the angles is equal to 180°.

Draw another triangle on the chalkboard and label the measures of two of the angels.

e.g.,



Ask the students how they would find the missing angle. (We add the two known angles and subtract from 180°.)

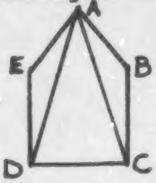
Repeat this procedure with other triangles.

7.09.07

7.09.07.01 **find the sum of the angle measures of a polygon**

STRATEGY 1: Draw a quadrilateral labeled ABCD on the chalkboard and say, "How can you change the quadrilateral into two triangles? (By drawing diagonal AC or DB we can change quadrilateral ABCD into two triangles, each having 180°.) Since quadrilateral ABCD has two triangles in it, it has a total of 360°. To determine the number of triangles formed in a polygon, diagonals must be drawn from one vertex." Draw a pentagon on the chalkboard.

e.g.,



Ask the students to start at vertex A and draw diagonals to C and D. Say, "How many triangles are formed? (3) Since each has 180°, pentagon ABCDE has 540°."

Repeat the procedure with a hexagon. The hexagon will form 4 triangles, each having 180° or a total of 720°. Ask the students to chart their findings.

e.g.,

	Number of sides	Sum of interior angles	Number of triangles formed
triangle	3	180°	1
rectangle	4	360°	2
pentagon	5	540°	3
hexagon	7	720°	4
octogon	8		
decagon	10		
dodecagon	12		

STRATEGY 1: cont'd

Say, "What happens to the sum of the interior angles as we add another side? (We also add 180° to the previous sum.) How can you figure out the sum of the angles of a 20 sided polygon?" Introduce the following formula.

e.g.,

$$S = (n - 2)180°$$

Say, "The sum of the interior angles of a polygon is equal to the number of its sides, take away 2, times 180°." Demonstrate that in a decagon of 10 sides, the sum of the angles is as follows.

e.g.,

$$\begin{aligned} S &= (n - 2)180 \\ S &= (10 - 2)180 \\ S &= (8)180 \\ S &= 1440° \end{aligned}$$

Ask the class to use the formula with numbers that they decide upon. Be sure to say that a polygon can have an infinite number of sides.

STRATEGY 2: Write the following formula on the chalkboard.

e.g.,

$$S = (n - 2)180°$$

Say, "This is the formula used to find the sum of the interior angles of a polygon. Can you guess what the letters S and N represent?" (S means sum and N means the number of the sides of the polygon.) Draw a picture of a triangle on the chalkboard. Ask how many sides there are in a triangle. (3) Demonstrate that the sum of the angles of a triangle is equal to 180°.

e.g.,

$$\begin{aligned} S &= (n - 2)180° \\ S &= (3 - 2)180° \\ S &= (1)180° \\ S &= 180° \end{aligned}$$

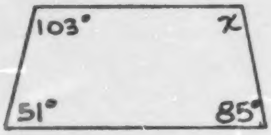
Repeat this procedure for quadrilaterals, pentagons, hexagons, octogons and decagons.

7.09.07

7.09.07.02 **find the sum of the angles of any polygon in numerical problems**

STRATEGY 1: Draw the following quadrilateral on the chalkboard.

e.g.,



Ask the class if they can figure out the missing angle. (First, we have to know how many degrees there are in the whole polygon.) Discuss the formula derived in P.O. 7.09.07.01, S = (N -2)180°. Say that in any quadrilateral there are 360°. Ask for a method to find the missing angle. (We can add up 51° + 103° + 85° and subtract from 360°.)

e.g.,

$360 - (51 + 103 + 85) = x$
$360 - 239 = x$
$121° = x$

Say, "I'm thinking of a triangle that has two angles of 45° each. What is the measure of the remaining angle?" Remind the students that a triangle has a sum of 180°. ($x = 180 - (45 + 45)$ or 90°) Ask the students to make up their own problems using different kinds of polygons.

STRATEGY 2: Write the following problem on the chalkboard.

> The interior angles of a certain polygon add up to 2,160°. How many sides does this polygon have?

Say, "How would one go about finding an answer?" Suggest the possibilities of using trial and error. (This method would take too long.) Ask if a chart can be used.

7.09.07

STRATEGY 2: cont'd

e.g.,

# of sides	shape	sum of interior ∢'s
3	△	180°
4	□	360°
5	⬠	540°
6	⬡	720°

Say that we can also use the formula, $S = (N-2)180$.

e.g.,

$$S = (N - 2)180$$
$$\frac{2,160}{180} = \frac{(N - 2)\ 180}{180}$$
$$12 = N - \cancel{2}$$
$$+2 \qquad \cancel{+2}$$
$$14 = N$$ (the polygon has 14 sides)

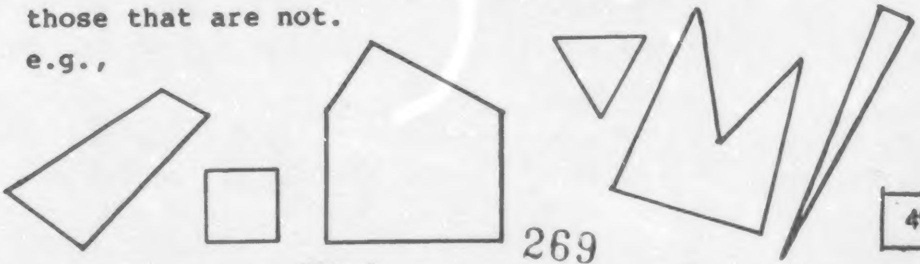
Repeat this procedure using the following polygons whose sums of interior angles are 3600°, 9180°, 11,340°.

7.09.07.03 identify a regular polygon (equilateral and equiangular)

STRATEGY 1: Say, "In a previous lesson we talked about different types of polygons and how to find the sum of their angles. But now we're going to talk about special polygons, whose angles and sides are all equal. These are called regular polygons. How can we identify regular polygons?" (They are all symmetrical.) Draw some examples. (△ equilateral triangle, □ square, ⬠ regular pentagon, ⬡ regular hexagon...)

Draw some polygons on the chalkboard and ask the students to identify those that are regular and those that are not.

e.g.,



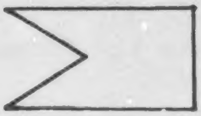
269

495

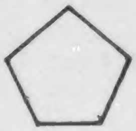
7.09.07

STRATEGY 1: cont'd
Ask the students to identify some things around us that are regular polygons, e.g., spider webs, the pentagon, flowers...

STRATEGY 2: Draw the following two pentagons on the chalkboard. e.g.,



I



II

Ask the class to describe each polygon. (Each figure is a polygon with 5 angles and 5 sides, however, figure II has 5 equal angles and 5 equal sides.) Say that a polygon that has all of its sides and angles equal is called a regular polygon. (You may want to add that all angles must be concave.) Say that figure II is called a regular pentagon. Ask for another name for a square. (regular quadrilateral) Ask the class to try drawing the following figures.

1) a regular hexagon
2) a regular octagon (stop sign)

Repeat this procedure for other regular polygons.

7.09.07.04 **find each interior angle of a regular polygon in numerical problems**

STRATEGY 1: Say, "If I had 360 dollars to divide equally among four students, how much would each one get? (90 dollars) How did you arrive at this answer? (360 ÷ 4 = 90) In a square there are 360° and each angle contains 90°. In a regular polygon, to find one of the angles, we divide the sum of all the angles by the number of angles." Ask the class to consider a 12 sided regular dodecagon. Say that since there are 12 sides and 12 angles, the sum of the angles

STRATEGY 1: cont'd

is 1800°.

e.g.,

$$S = (12 - 2)180$$
$$S = (10)180$$
$$S = 1800^\circ$$

Say, "To find the number of degrees in each of the 12 angles, we divide by 12."

e.g.,

$$\begin{array}{r} 150^\circ \\ 12\overline{)1800} \end{array}$$

Ask the students to find the number of degrees in each angle of a regular square, pentagon, hexagon, octagon and decagon.

STRATEGY 2: Draw a square on the chalkboard. Ask a student to name this figure. (a square or a regular quadrilateral) Ask the class to find the sum of the interior angles using the formula $S = (N - 2)180^\circ$. (The sum of the interior angles is 360°.) Say, "If the sum of all four angles is 360°, how many degrees are in each angle? (Since each angle is equal, because this is a regular polygon, we divide 360° by 4 to get 90° in each angle.) To find the number of degrees in an interior angle of a regular polygon, we find the sum of the interior angles using the formula $S = (N - 2)180^\circ$. Then we divide by the number of sides in the polygon."

Write this example on the chalkboard.

e.g.,

Find the number of degrees in angle x if this is a regular hexagon.

1) $S = (N - 2)180$
$S = (6 - 2)180$
$S = (4)180$
$S = 720$

2) $$\begin{array}{r} 120 \\ 6\overline{)720} \end{array}$$
$x = 120^\circ$

Repeat this procedure using a pentagon, decagon and dodecagon.

P.O. 7.09.07.01

NAME: ____________________ CLASS: __________ DATE: __________

Complete the chart.

# sides of a polygon	# of triangles formed	Sum of the angles
a. 7		
b. 19		
c. 36		
*d.		900°

P.O. 7.09.07.02

NAME:______________________________CLASS:__________DATE:________

1. In each of the following, find the measure of the fourth angle of a quarilateral when the other 3 angles measure

a. 55°, 81°, and 103°

b. 119°, 90°, and 90°

c. 115°, 14°, and 152°

2. In each of the following, find the measure of the fifth angle of a pentagon when the other 4 angles measure

a. 90°, 90°, 60°, 150°

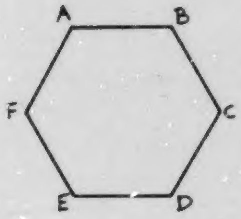
b. 83°, 165°, 75°, 175°

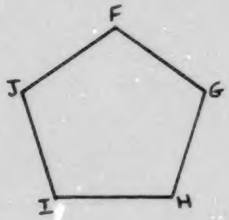
c. 180°, 55°, 120°, 97°

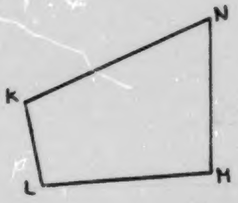
P.O. 7.09.07.03

NAME:______________________________ CLASS:__________ DATE:________

Identify each regular polygon.







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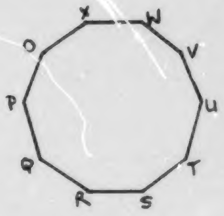
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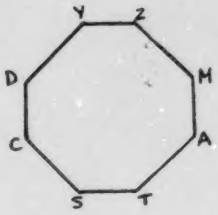

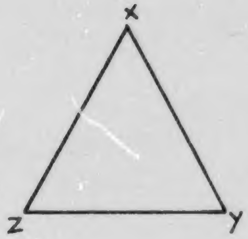

_______________ _______________ _______________

500

P.O. 7.09.07.04

NAME: ______________________________ CLASS: __________ DATE: ________

A. Find the sum of the interior angles and each interior angle for a regular polygon of

a. 9 sides

b. 10 sides

B. What steps would be used to find the number of degrees in the sum of the angles of any polygon? In each interior angle of a regular polygon?

501

Unit 55

7.01.07 SETS

(3 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

7.01.07.01A — use set notation to list the members of a set from a description of the set;
e.g.,
{The days of the week} =
{Sun., Mon., Tues., Wed., Thurs., Fri., Sat.}

*7.01.07.02A — identify as a finite set a given set with a specific number of elements;
e.g.,
{the set of vowels}

*7.01.07.03A — identify as an infinite set a given set with an unlimited number of elements;
e.g.,
{The set of whole numbers}

7.01.07.04A — determine whether or not two sets are equal or equivalent;
e.g.,
{1,2,3,4} and {6,7,8,9} are equivalent sets.
{8,9,5,4} and {9,5,4,8} are equal sets

*7.01.07.05A identify an empty set or null set as a given set with no elements;

e.g.,

{ } is the empty set or the null set. Another symbol for the empty set is "Ø".

7.01.07.06A state the definition of consecutive numbers, even numbers and odd numbers in word problems.

e.g.,

Select any two odd numbers. Is their sum an odd number or an even number?

{1, 3, 5, 7....}
1 + 5 = 6
6 is an even number.

Select any two even numbers. Is their sum an odd number or an even number?

{2, 4, 6, 8,...}
2 + 6 = 8
8 is an even number.

Select any even and any odd number. Is their sum an odd number or an even number?

{1, 2, 3, 4, 5,...}
2 + 5 = 7
7 is an odd number.

7.01.07

VOCABULARY

TERMS	SYMBOLS
elements	
set	
collection	
set brackets	{ }
well defined set	
finite set	
infinite set	
equal sets	
equivalent sets	
null or empty set	∅, { }

GETTING STARTED

Cut out pictures from magazines and newspapers of trees, cars, numbers, birds, animals... Tape them on pieces of construction paper in sets. Say, "Look at the collection of things on this paper. Each collection is known as a set. There is a set of ____ on this paper. What other sets do you see? (set of trees, set of numbers, set of birds, set of animals...) There are sets of things in our classroom, e.g., the set of children, the set of desks and the set of chairs. Write the names of as many other sets in the room as you can."

Say, "Can you find a set of tall children in the classroom?" (The students should be uncertain as to what constitutes "tall.") Say that the description of the set is too vague and unclear by picking a student of average height and saying, "Is he/she in the set? Is it difficult to be sure? (yes) Why? (The set is not well defined.) Is the set described by the words, the set of pets of students in our class, clear or unclear? Why? (Clear, because it is well defined.) Does anyone have a dog (cat, canary...)?" Survey the pets belonging to the students in the class, and write each pet on the chalkboard. When the survey is finished, say, "Brackets are usually put around a set like this." Write {dog, cat, turtle, fish, bird, hamster} . Say, "This is read as the set whose members are dog, cat, turtle, fish, bird and hamster."

7.01.07

7.01.07.01 **use set notation to list the members of a set from a description of the set**

STRATEGY 1: Say, "There are five boroughs in New York City. Who can name them? Now, write this set using set brackets: The set whose members are the boroughs in New York City. ({Manhattan, The Bronx, Queens, Brooklyn, Staten Island}) A set is any collection of objects. These objects are called the members or elements of the set. What are the members or elements of the set of whole numbers less than 10? We list them by using set brackets. ({0, 1, 2, 3, 4, 5, 6, 7, 8, 9}) Does 2 belong to this set? (yes) Does 10? Does $\frac{1}{2}$? A set may be specified by listing the category of all its elements, or by giving a description of its elements. It is usual to name a set by using a capital letter. If Set A is the set of whole numbers that are less than five, then A = { 0, 1, 2, 3, 4}. What are the elements of Set B, if Set B is the set of whole numbers that divide 6 evenly?" (B = {1, 2, 3, 6}) Write both sets on the chalkboard.

Repeat this procedure finding and listing other sets.

STRATEGY 2: Say, "Which months have 31 days?" Ask a student to say the answers as you write them on the chalkboard. (January, March, May, July, August, October, December) Say, "Write these seven months as a set using set brackets. ({January, March, May, July, August, October, December}) A set is any collection of objects. These objects are called the members or elements of the set. What are the members or elements of the set of the days of the week? Write them as a set, using set

STRATEGY 2: cont'd

brackets. ($\{$Saturday, Sunday, Monday, Tuesday, Wednesday, Thursday, Friday$\}$) Write the set whose members are United States coins whose value is less than 50 cents. ($\{$penny, nickel, dime, quarter$\}$) A set may be specified or described by listing the members or elements, or by describing them. Usually sets are named by using a capital letter. For example, if Set B is the set of prime numbers less than 10, then $B = \{2, 3, 5, 7\}$. Set C is the set of whole numbers that divide into 24 evenly. Write set C." ($C = \{1, 2, 3, 4, 6, 8, 12, 24\}$)

Repeat this procedure by writing other sets and asking students to label them using capital letters.

7.01.07.02 **identify as a finite set a given set with a specific number of elements**

STRATEGY 1: Prepare 22 index cards on which are written the names of the 22 elements of the following four sets.

a) days of the week	(7 cards)
b) boroughs of New York City	(5 cards)
c) whole numbers between 4 and 10	(5 cards)
d) vowels	(5 cards)

Prepare four pieces of construction paper (one for each set) with the name of the set written on each. Allow students to pick a card and tape it to the appropriate paper. Continue until all of the cards have been picked. Say to the class that a set whose elements can be counted is a finite set. (The sets above are finite sets.)

STRATEGY 2: Ask the students to name the elements of each set

STRATEGY 2: cont'd

described below.

a) the months of the year that end in ber
b) the numbers on the face of a clock
c) the whole numbers between 3 and 9

Ask the students to write their answers as indicated below.

a) {September, October, November, December}
b) {1, 2, 3, 4, 5, 6, 7, 8, 9, 10 11, 12}
c) {4, 5, 6, 7, 8}

Say to the class that we call a set whose elements can be counted, a finite set. Say that the sets above are finite sets.

7.01.07.03 **identify as an infinite set a given set with an unlimited number of elements**

STRATEGY 1: Prepare index cards numbered $\frac{1}{2}$, 1, $1\frac{1}{2}$, 2, $2\frac{1}{2}$, 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, 5, $5\frac{1}{2}$, 6, $6\frac{1}{2}$, 7, $7\frac{1}{2}$, 8, $8\frac{1}{2}$, 9, $9\frac{1}{2}$ and 10. Prepare a piece of construction paper labeled "The Whole Numbers." Ask various students to pick a card and decide whether or not to tape it onto the paper. Continue until all of the cards have been picked. Say, "Can we include any other numbers? (The set of whole numbers has more elements than have been shown.) How many more index cards do we need to completely list all of the elements in this set? (The set is unlimited or infinite.) If the members of a set cannot be counted with the counting coming to an end, the set is called an infinite set."

STRATEGY 2: Ask the students to name the elements of the set described below, e.g., "The set of whole numbers". Ask the students to write their answers as follows, e.g., {0, 1, 2, 3, ...}.

STRATEGY 2: cont'd Say that the three dots are used to indicate that there is no end to the listing of the members of the set. Say that if the members of a set cannot be counted with the counting coming to an end, the set is called an infinite set. Say, "Does this set end? Why not?" (The set of whole numbers is an infinite set.)

7.01.07.04 **determine whether or not two sets are equal or equivalent**

STRATEGY 1: Prepare ten index cards on which are written the whole numbers 1 through 10. Ask the students to form the following two sets using the cards. e.g.,

a) the set of odd numbers from 1 to 10
b) the set of even numbers from 1 to 10

Say that an even number is a whole number which can be divided by two with no remainder and that an odd number is any whole number that is not divisible by two with no remainder.

Say that two sets are called equal sets if they contain the same elements. Say, "Why isn't {1, 2, 5, 7, 9} equal to {2, 4, 6, 8, 10}?" (The elements of one set are not the same as the elements of the other.)

Say that two sets are called equivalent sets if they have the same number of elements. Say that the two sets above are equivalent. Ask a student to tell how many elements are in each set. Write the definitions of equal sets and equivalent sets on the chalkboard.

Repeat this procedure using other pairs of sets.

7.01.07

STRATEGY 2: Ask the students to list the elements in the following sets.
e.g.,

a) even numbers from 9 through 21
b) odd numbers from 4 through 16

Say that the two sets are called equal sets if they contain the same elements. Say, "Why isn't $\{10, 12, 14, 16, 18, 20\}$ equal to $\{5, 7, 9, 11, 13, 15\}$?" (The elements of one set are not the same as the elements of the other set.) Say that the two sets are called equivalent sets if they have the same number of elements. Say, "Are these sets equivalent?" (The two sets above are equivalent because they have the same number of elements.)

Repeat this procedure using other pairs of sets.

7.01.07.05 **identify an empty set or null set as a given set with no elements**

STRATEGY 1: Prepare 10 index cards on which the numbers 1 through 10 are written. Say, "Which elements fit into these sets?"

a) the set of whole numbers between 5 and 6
b) the set of even numbers between 6 and 8
c) the set of odd numbers between 3 and 5

Elicit that there are no appropriate numbers for any of these sets. Define a set with no elements as the empty or null set. Ask the students to write their answers in either form, e.g., $\{\}$ or $\emptyset$.

Repeat this procedure using other empty sets.

STRATEGY 2: Ask the students to list the elements in the following sets.

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STRATEGY 2:
cont'd

a) the days of the week that begin with a vowel
b) the set of odd numbers that are divisible by 2 with no remainder
c) the set of even numbers between 6 and 8

Elicit that these sets have no members. Explain that a set with no elements is called an empty or null set. Write $\{\}$ and $\emptyset$ on the chalkboard. Ask the students to write their answers in either form $\{\}$ or $\emptyset$.

7.01.07.06 **use the definition of consecutive numbers, even numbers, and odd numbers in word problems**

STRATEGY 1: Prepare index cards numbered from 1 through 10 and ask the students to show sets of five consecutive numbers, e.g., $\{3, 4, 5, 6, 7\}$. Say, "What is the definition of consecutive numbers?" (Two whole numbers are consecutive if their difference is one.) Ask the students to place the index cards in sets to form six sets of five consecutive numbers not exceding ten.

e.g.,
$\{1, 2, 3, 4, 5\}$ $\{2, 3, 4, 5, 6\}$ $\{3, 4, 5, 6, 7\}$
$\{4, 5, 6, 7, 8\}$ $\{5, 6, 7, 8, 9\}$ $\{6, 7, 8, 9, 10\}$

Separate the cards into even and odd number piles. Ask a student to pick any two odd number cards. Say, "What is the sum of the numbers? Is it even or odd? What is the product? Is it even or odd?" Ask a student to pick any two even number cards. Say, "Is their sum even or odd? Their product?" Continue by asking the student to select any even and any odd number card. Say, "Is their sum even or odd? Their product?" Repeat this procedure until all of the cards have been used.

STRATEGY 2: Ask the students what consecutive numbers mean. (Two whole numbers are consecutive if their difference is one.) Say, "How many sets of three consecutive numbers are possible, using the whole numbers from 1 to 10?" (Eight possible sets of consecutive numbers.)

e.g.,

{1, 2, 3}	{2, 3, 4}	{3, 4, 5}	{4, 5, 6}
{5, 6, 7}	{6, 7, 8}	{7, 8, 9}	{8, 9, 10}

Say, "Should {9, 8, 7} be added to the list? ({9, 8, 7} and {7, 8, 9} contain the same elements, therefore, {9, 8, 7} is already listed.) When two numbers are consecutive, must one of them be even? Must one be odd?" (One more than an even number makes an odd number, and one more than an odd number makes an even number. Therefore, the answer is yes to both questions.)

Ask each student to select any two odd numbers. Say, "Is their sum an odd number or an even number? Is their product an odd or even number?" Repeat this procedure for two even numbers. Repeat this procedure for any odd and any even number.

Ask the students to find the sum of the first

a) two odd numbers (4);
b) three odd numbers (9);
c) four odd numbers (16).

Say that the sum of odd whole numbers can be found by squaring the amount of odd numbers being added. Say, "What is the sum of the first 9 odd numbers?" (The sum is 9 squared or 81.)

P.O. 7.01.07.01

NAME: ______________________________ CLASS: __________ DATE: ________

List the members of each set described below.
Write your answers in set notation.

1. {The months with 31 days}
2. {Factors of 6}
3. {Odd numbers between 36 and 42}
4. {Prime numbers between 36 and 42}
5. {Whole numbers less than 25 that are divisible by 4}
6. {The months in the year that begin with the letter F}
7. {Even numbers between 1 and 11}
8. {The states in the United States completely surrounded by water}
9. {Prime factors of 30.}
10. {Whole numbers greater than 20 and less than 26}

P.O. 7.01.07.02-03

NAME:____CLASS:__________DATE:________

I. Identify each of the following sets as being finite or infinite.
II. Use set notation to list the members of each set. Use the three dots wherever necessary.

I.

1. The set of all the grains of sand on a given beach. ________

2. The set of all even numbers greater than 10. ________

3. The set of all fractions between the numbers 8 and 9. ________

4. The set of all people now alive on earth. ________

5. The set of whole numbers greater than 5. ________

6. The set of integers less than 5. ________

7. The set of whole numbers from 1 to 10,000. ________

II.

1. 1. {All even whole numbers} = ____________________________

2. {All whole numbers} = ____________________________

3. {All odd whole numbers} = ____________________________

4. {All odd whole numbers up to 3000} = ____________________________

5. {All fractions greater than 1 and less than 2 whose denominator is 17} = ____________________________

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P.O. 7.01.07.04.-05

NAME:____________________ CLASS:________ DATE:________

I. In the blanks below, write the letter of the set in column B which is equal to the set after the blank in column A.

II. In the blanks below write the letter of the set in column B which is equivalent to the set after the blank in column A.

I.

Column A	Column B
____1. $\{3, 15, 8, 12\}$	A. $\{12, 8, 15, 3\}$
____2. {The set of whole numbers greater than 10 and less than 5}	B. $\{13, 17, 19\}$
____3. {The set of all three digit numbers}	C. $\{2, 3\}$
____4. $\{\frac{1}{4}, \frac{1}{2}, \frac{3}{4}\}$	D. $\{0, 3, 6, 9\}$
____5. {Prime numbers between 12 and 20}	E. $\emptyset$
____6. {Prime factors of 6}	F. $\{20, 19, 18, 17\}$
____7. {Digits in the number 321}	G. $\{5, 6, 7\}$
____8. {Whole numbers divisible by 3}	H. $\{100, 101, 102, \ldots, 999\}$
____9. $\{\frac{20}{4}, \frac{30}{5}, \frac{42}{6}\}$	I. $\{1, 2, 3\}$
____10. $\{1, 2, 3, 4, \ldots, 20\}$	J. $\{0.25, 0.5, 0.75\}$

II.

Column A	Column B
____1. $\{4, 5, 6\}$	A. {The set of legs on a normal dog}
____2. {A, B, C, D, E, F, G}	B. $\{2, 3, 5, 7, 11, 13, 17\}$
____3. $\{\frac{7}{8}, \frac{8}{9}, \frac{9}{10}, \frac{10}{11}\}$	C. $\{10, 20\}$
____4. {Wheels on a bicycle}	D. {A, B, C}

P.O. 7.01.07.06

NAME:______________________________________CLASS:________DATE:_______

Tell whether the following sentences are true or false. Correct any false statement to make it true.

1. There is an odd number between every pair of even numbers.

2. There is an even number between every pair of odd numbers.

3. One more than any even number makes an odd number.

4. One more than any odd number makes an even number.

5. The sum of any two odd numbers is an even number.

6. The product of any two odd numbers is an even number.

7. The sum of any odd and any even number is an even number.

8. The product of any odd and any even number is an even number.

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7.01.08 RELATIONSHIPS

(4 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

7.01.08.01A determine subsets of a given set;

e.g.,

$\{R, W\}$ is a subset of

$\{A, Q, R, M, W\}$ We write:

$\{R, W,\} \subset \{A, Q, R, M, W\}$

7.01.08.02B determine the number of subsets of a finite set of up to eight elements using the expression 2^n, where n is the number of elements in the set;

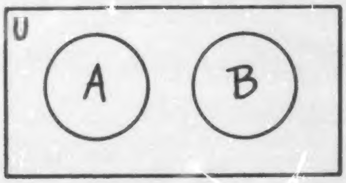
e.g.,

$\{\emptyset , a, Q\}$

The number of subsets of this set are 2^3 or 8.

*7.01.08.03A represent two disjoint sets with a Venn diagram;

e.g.,

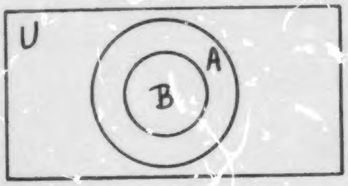


A and B are disjoint sets.

*7.01.08.04A

represent a set and a subset with a Venn diagram;

e.g.,



B is a subset of A.

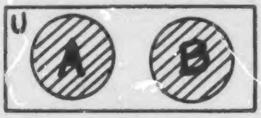
*7.01.08.05A

represent and list the union of two given sets on a Venn diagram;

e.g.,

A = {2, 8, 12}

B = {6, 5, 7}



A ∪ B = {2,6,7,8,12}

*7.01.08.06A

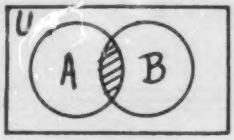
represent and list the intersection of two given sets on a Venn diagram;

e.g.,

A = {△, ø, ▭}

B = {▭, ø, *}

A ∩ B = {ø, ▭}



*7.01.08.07A

find the Least Common Multiple (LCM) of the elements of two given sets of multiples.

e.g.,

A = {6, 12, 18, 24,...} = multiples of 6

B = {8, 16, 24,...} = multiples of 8

A ∪ B = {24, 48,...} = multiples of both 6 and 8.

Least Common Multiple = {24}

7.01.08

VOCABULARY

TERMS

subset
disjoint
Venn diagram
union
intersection
S

GETTING STARTED

Write on the chalkboard, "Express the following in set notation."

a) The set of days in the week beginning with "S".
b) The set of vowels in the alphabet.
c) The set of months beginning with "J".
d) The set of symbols used in Roman numbers.
e) The set of symbols used in operations with numbers.
f) The set of even numbers less than twenty.

Ask a student if any of the sets have the same number of members (elements). Say that two sets which have the same number of elements are equivalent. Say, "How many elements are in the set of vowels? ...set of symbols used in operations with numbers? The sets a to f all consist of elements which terminate or end, and these sets are called finite sets."

Write the following on the chalkboard.

> Steve and Joe were sitting in the park. They decided to name the cars as they passed. They saw the following cars: Buick, Toyota, Firebird, Datsun, Chevrolet, Honda, Ford and Mercedes.

Ask the class to express in set notation the foreign cars named. Ask a student to come to the chalkboard and write the set.
e.g.,

C = {Toyota, Datsun, Honda, Mercedes}

Repeat this procedure using other sets.

7.01.08

7.01.08.01 determine the subsets of a given set

STRATEGY 1: Ask the students to write in set notation the first ten letters of the alphabet.

e.g.,

$$A = \{a,b,c,d,e,f,g,h,i,j\}$$

Ask a student to name the vowels in this set, e.g., $V = \{a,e,i\}$. Say that set V is a subset of set A. Write on the chalkboard how this is written in set notation.

e.g.,

$$V \subset A$$

Explain that the symbol $\subset$ means "is a subset of". Ask the students to create and label other subsets of A.

Ask the students to create a set and label it. Ask them to write the subset of their sets using the symbol $\subset$.

STRATEGY 2: Write the following set on the chalkboard.

e.g.,

$$B = \{1,2,3,4,5,6,7,8,9,10\}$$

Ask a student to name the prime numbers in set B, e.g., 2,3,5,7, and label this set P. Explain that set P is a subset of set B. Say, "A subset is a set of elements from the original set." Write the following on the chalkboard.

e.g.,

$$P \subset B$$

Repeat this procedure asking the following directions.

e.g.,

a) Write the subset of numbers divisible by 3. Write them in set notation.

b) Write the subset of numbers divisible by 2.

c) Write the subset of numbers divisible by 1.

Repeat this procedure forming other subsets of set B.

7.01.08

7.01.08.02 determine the number of subsets of a finite set of up to eight elements using the expression 2^n where n is the number of elements in the set

STRATEGY 1: Construct the following chart on the chalkboard.

e.g.,

Set	Number of Elements of the set	Number of Subsets
1) a		
2) Judy, Sara		
3) 4,5,6		
4) Dave, Joe, Steve, Sam		
5) penny, nickel, dime, quarter, half dollar		

Ask the students to fill in the number of elements in each set. Ask the students to name the subsets of each set. Explain that a subset of every set is the empty set, e.g., $\{\}$ or $\emptyset$. Name the subsets of set 4 on the chart. Explain that a subset of every set is the set itself. Say that each element of a set is a subset of the original set. After the chart is filled in, ask the students if there is a relationship between the number of elements in a set and the number of subsets in a set.

a) Set 1 has one element and two subsets;

b) Set 2 has two elements and four subsets.

Repeat this procedure with the other sets. Ask the students if they can determine the number of subsets in a set with five elements, six elements, seven elements, eight elements.

Identify and write the formula for determining the number of subsets of a finite set, $S = 2^n$ where n is the number of elements in the set.

STRATEGY 1: cont'd

e.g.,

$P = \{4,5,6\}$ $n = 3$

$$S = 2^n$$
$$S = 2^3$$
$$S = 2x2x2$$
$$S = 8$$

Use this formula to determine the number of subsets in sets 1 through 8.

STRATEGY 2: Write the following problem on the chalkboard.

> You want to make up signals with a friend who lives across the street. How many different signals are possible if you can use any of 3 different colors of paper: red, white and blue?

(There are at least three possibilities.) Write each in set notation on the chalkboard.

e.g.,

a) 3 papers {blue} , {red} or {white}
b) 2 papers {red, white} or {red, blue}
c) 1 paper {red, white, blue}

Say, "A fourth possibility exists, e.g., a signal with no paper. A set with no elements is called an empty set." Write the symbol { } or Ø on the chalkboard.

Ask the students to list all the subsets for the problem above. Say that each element of a set is a subset of the original set.

Ask the students to make a generalization about the relationship between the number of elements in a set to the number of subsets of a set. ($S = 2^n$ where n equals the number of elements in the set.) Ask a student to determine the number of subsets in a set containing five elements.

e.g.,

$$S = 2^5$$
$$S = 2 \times 2 \times 2 \times 2 \times 2$$
S = 32 subsets

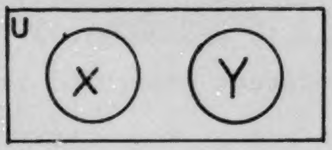
7.01.08

7.01.08.03 **represent two disjoint sets with a Venn diagram**

STRATEGY 1: Say, "A picture is worth a thousand words. A Venn diagram is a picture of set relationships. Venn diagrams were named in honor of an English mathematician named John Venn." Construct a Venn diagram on the chalkboard for the following.

e.g.,

x = {the set of boys in your class}
y = {the set of girls in your class}



Say that the rectangle represents the Universal Set (U). Say, "Since the sets x and y have no elements in common, they are called disjoint sets. They are shown as two separate circles in a universal set."

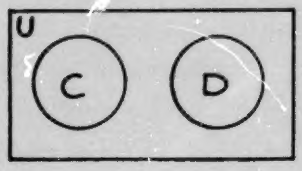
Repeat this procedure with other disjoint sets. Ask the students to draw Venn diagrams of the following.

1) G = the set of General Motors cars
   F = the set of Ford Motor cars

2) E = the set of even numbers less than 9
   O = the set of odd numbers less than 9

STRATEGY 2: Use oak tag to construct and display the following diagram.

e.g.,



Say, "C equals the set of triangles, D equals the set of rectangles and U represents the Universal Set of all plane figures. This diagram is called a Venn Diagram. This is a 'picture' of a set

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STRATEGY 2: cont'd

relationship. Do set C and set D have any elements in common? (The sets C and D have no elements in common.) Sets that have no elements in common are called disjoint sets are shown as separate circles."

Ask the students to construct Venn diagrams of the following pair of sets.

A = {Judy, Sara, Maria}
B = {Steve, Dave, Jose}

Ask, "Does set A have elements that are also elements of set B? Does set B have elements that are also elements of set A?"

Repeat this procedure with other disjoint sets.

7.01.08.04 represent a set and a subset with a Venn diagram

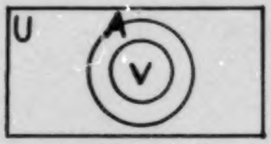
STRATEGY 1:

Write the following on the chalkboard.

e.g.,
A = {the set of letters in the alphabet}
V = {the set of vowels}

Say, "The set of vowels is a subset of the set of letters in the alphabet. In set notation it is written as $A \subset V$. We can show this in a Venn diagram as follows."

e.g.,



Repeat this procedure with these sets.

e.g.,
C = {1,2,3,4,5}
D = {2,3}

Ask the students to diagram the following.

a) S = {the set of squares}
R = {the set of rectangles}

7.01.08

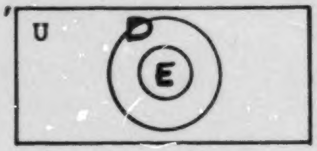
STRATEGY 1: cont'd

b) N = {9, 10}
M = {8,9,10,11}

c) A = {1,3,7}
D = {1,3,5,7}

STRATEGY 2: Draw the following on the chalkboard.

e.g.,



Say that this is a Venn diagram of two sets, D and E, contained in the universal set (U).

e.g.,
D = {Dorothy, Jessie, Ben}
E = {Dorothy, Jessie}

Ask the students why set E is inside set D. (Set E is a subset of set D.) Write the following on the chalkboard.

e.g.,
J = {the set of baseball players on the Yankee team}
M = {Winfield, Nettles, Righetti}

Ask the students to construct their own diagrams of these sets.

7.01.08.05 **represent and list the union of two given sets on a Venn diagram**

STRATEGY 1: Write on the chalkboard.

A = {a,b,c}
B = {d,e,f}

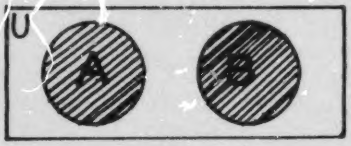
Ask the students what the elements would be if we joined the two sets. (a,b,c,d,e,f) Say, "We call the joining of these sets the union of sets A and B. The symbol $\cup$ represents union, e.g., A $\cup$ B is the union of set A and set B."

7.01.08

STRATEGY 1: cont'd

Say that the "union" of two sets may be shown in a Venn diagram.

e.g.,



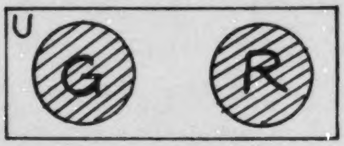
Say, "We show the union of two disjoint sets by shading A and B."

Write the following on the chalkboard.

> It is opening day of the National Football League at the Meadowlands with the Giants playing the Washington Redskins. The Giant players are introduced and line up on the field. The Redskin players are introduced and line up opposite the Giants. The band plays the Star Spangled Banner.

Ask the students to draw a Venn diagram to represent the union of the two teams.

e.g.,



G = { the Giants }
R = { the Redskins }
G ∪ R = { Giants and Redskins }

Say that the teams have no players in common.

Say that there is another way to show the union of two sets where the sets have one or more elements in common. Write these sets on the chalkboard.

e.g.,

G = {the set of students in your class}
H = { the set of students wearing sneakers}
G ∪ H ={the set of students in your class and the students who are wearing sneakers }

Ask all the students in the class to raise their

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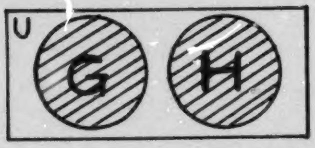
STRATEGY 1: cont'd

hand. Ask all of the students who are wearing sneakers to stand. Say that the students who are standing with their hands raised are called the union of the two sets because they are in both sets. Draw the Venn diagram to represent this.

e.g.,



Repeat the construction and listing of the union of the following sets.

1) $A = \{d,e,f,g\}$
$B = \{a,g,f,h\}$
$A \cup B = \{d,e,f,g,a,h\}$

2) $x = \{3,6,12\}$
$y = \{4,8,12\}$
$x \cup y = \{3,4,5,6,8,12\}$

STRATEGY 2: Write the following problem on the chalkboard.

> There are 12 students in the band and 10 students in the chorus. What is the total number of students in both groups?

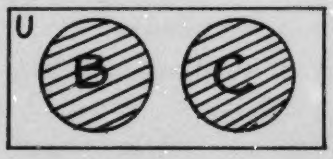
Say that this problem can be shown by constructing a Venn diagram for the sets. (Note: letters represent students)

e.g.,

$B = \{A,B,C,D,E,F,G,H,I,J,K,L\}$
$C = \{M,N,O,P,Q,R,S,T,U,V\}$

Write $B \cup C = \{A,B,C,D,E,F,G,H,I,J,K,L,M,N,O,P,Q,R,S,T,U,V\}$ on the chalkboard. Say that the symbol "   " means the "union" of the two sets, e.g., set B and set C put together. Say, "This union can also be shown in a Venn diagram."

e.g.,



STRATEGY 2, cont'd

Say that the union of two disjoint sets is shown by shading the sets. Say, "The sets are disjoint since they have no common members or elements."

Say, "Sometimes sets have one or more elements in common, and the union of these sets is shown in another way. The band and chorus have both acquired two new members, W and X. W and X are elements in both the band and chorus."

e.g.,

$$B = \{A,B,C,D,E,F,G,H,I,J,K,L,W,X\}$$
$$C = \{M,N,O,P,Q,R,S,T,U,V,W,X\}$$

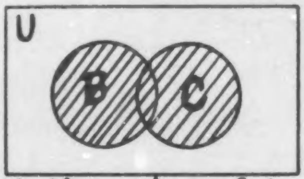
Ask the students to write in set notation the union of the two sets.

e.g.,

$$B \cup C = \{A,B,C,D,E,F,G,H,I,J,K,L,\ M,N,O,P,Q,R,S,T,U,V,W,X\}$$

Say, "How many times did W and X appear?" (Only once since they are common to both sets.) Draw the Venn diagram on the chalkboard.

e.g.,



Say that the union of two sets are the elements that they have in common and all of the other elements.

7.01.08.06 **represent and list the intersection of two given sets on a Venn diagram**

STRATEGY 1: Write on the chalkboard.

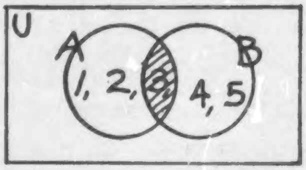
$$A = \{1,2,3\}$$
$$B = \{3,4,5\}$$

Say, "Are there any members of set A that are also in set B? (3) We say that $\{3\}$ is the intersection of A and B and we write it like this." Write $A \cap B$

STRATEGY 1: cont'd

on the chalkboard. Say, "The sentence A $\cap$ B = $\{3\}$ means that set A and set B have one element in common and that element is 3." Draw a Venn diagram that will represent the intersection of sets A and B.

e.g.,



Say, "We shade the intersection of the sets. The intersection of two sets helps us find a third set that includes only the common elements of both sets."

Ask the students to list and diagram the intersection of the following sets.

1) x = { a,b,c,d,e }
   y = { a,e,i,o,u }
   x $\cap$ y = {   }

2) P = {prime numbers less than 10}
   O = {odd numbers less than 10}
   P $\cap$ O = {   }

STRATEGY 2: Ask the students to list the following in set notation.

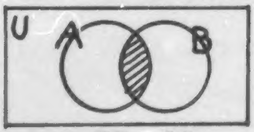
A) multiples of 3 to 30
B) multiples of 4 to 32

Ask two students to come to the chalkboard and write the set notations.

e.g.,
A = {3,6,9,12,15,18,21,24,27,30}
B = {4,8,12,16,20,24,28,32}

Say that each set can be drawn as a circle containing the elements listed within. Ask if any elements are common to both sets. (12 and 24) Explain that we can list and diagram these elements as a third set of elements common to both sets.

e.g.,



7.01.08

STRATEGY 2: cont'd

Say, "The shaded portion on the Venn diagram shows the intersection of the two sets." Write the symbol "   " on the chalkboard. Say, "This is the symbol used to show intersection in set notation." Ask a student to write the sentence which shows the intersection on the Venn diagram.

e.g.,

$$A \cap B = \{12,24\}$$

Repeat this procedure using other sets with common elements.

7.01.08.07 **find the Least Common Multiple (LCM) of the elements of two given sets of multiples**

STRATEGY 1:

Ask the students to write the multiples of 2 to 20 and the multiples of 5 to 20 in set notation.

e.g.,

$$A = \{2,4,6,8,10,12,14,16,18,20\}$$
$$B = \{5,10,15,20\}$$

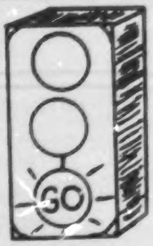
Say, "What is the intersection of A and B? ($A \cap B = \{10,20\}$) Since 10 is the smallest or least number in the set of common multiples, it is called the Least Common Multiple or LCM."

Repeat this procedure with other pairs of numbers. Say that the LCM can be helpful in determining the least common denominator in the addition and subtraction of fractions.

P.O. 7.01.08.01

NAME: ______________________________ CLASS: __________ DATE: ________

Answer the questions using set A.



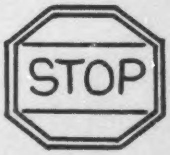
A. $0 = \{1,3,5,7,9,11\}$

1. What is the subset of all numbers divisible by 3?

2. What is the subset of all prime numbers?

3. What is the subset of numbers divisible by 1?

4. What is the subset of numbers divisible by 2?



P.O. 7.01.08.02

NAME: ____________________ CLASS: ________ DATE: ________

Use the formula 2^n to find the number of subsets in a given set.

Set	Number of Elements in the Original Set	Number of Subsets
T= 0, 1		
X= Judy, Sara, Maria		
K= a, b, c, d, e		
M= 1, 3, 5, 7		

* How many subsets in a set with 6 elements, 7 elements?

**How many subsets in the set of counting numbers?

P.O. 7.01.08.03

NAME: ________________________________ CLASS: ________ DATE: ________

Determine which are disjoint sets. Draw a Venn diagram to represent one of the disjoint sets.

Draw a Venn diagram to represent one of the disjoint sets.

1. X = {1, 3, 5, 7} Y = {2, 4, 6, 8} ________

2. A = {3, 6, 9, 12} B = {6, 8, 12, 18} ________

3. S = {a, b, c,} T = {x, y, z} ________

4. N = {Sara, Jane, Steve} M = {Jose, Robert, Steve Dave} ________

5. C = {The set of triangles} R = {The set of rectangles} ________

6. A = {The set of counting numbers} I = {The set of negative integers} ________

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P.O. 7.01.08.04

NAME: ______________________ CLASS: ________ DATE: _______

Write a subset for each set. Draw a Venn diagram to show each set and subsets.

Set	Subset	Venn Diagram
1. A= Maria, Judy, Yvette, Susan		
2. B=		
3. X=		
4. C= red, white, blue. green, black		
5. T= The New York Yankee's		

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P.O. 7.01.08.05

NAME: ____________________ CLASS: ________ DATE: ________

Use Venn diagrams to show union of two sets and list the union in set notation.

1.

S = {s, a, n, d}

B = {b, o, x}

2.

X = {h, o, r, s, e}

Y = {b, a, c, k}

3.

L = {blue, brown, red, black}

Q = {white, yellow, red, green}

4.

Make your own pair of sets.

P.O. 7.01.08.06

NAME:______________________________CLASS:________DATE:_______

Use a Venn Diagram to express the intersection of two sets. List the intersection using the symbol N.

1. C= {h,i,s} D= {s,t,o,r,y}	2. E= {m,i,s,t} F= {t,a,k,e}

3. Make your own two sets.

P.O. 7.01.08.07

NAME:______________________________CLASS:__________DATE:________

Find the "Least Common Multiple" (L.C.M.)

1. X = {3, 6, 9, 12 ...}

Y = {4, 8, 12 ...}

X ∪ Y = {   }

L. C. M. = {   }

2. A = {8, 16, 24 . . . }

B = {12, 24 ... }

A ∪ B = {   }

L. C. M = {   }

3. M = {4, 8, 12, 16 ...}

N = {8, 16 . . . }

M ∪ N = {   }

L. C. M. = {   }

4. S = {12, 24}

T = {10, 20}

S ∪ T = {   }

L.C.M.= {   }

Unit 57

7.01.09 PRIME AND COMPOSITE NUMBERS

(2 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

7.01.09.01A determine whether a number is prime or composite;

e.g.,

13, 41 are prime numbers

48, 32, 14 are composite numbers

7.01.09.02A list and order the prime factors of a number;

e.g.,

The prime factors of 60 are 3x2x2x5.

In order, these factors are 2x2x3x5.

7.01.09.03A express prime factors in exponential notation.

e.g.,

$$
\begin{aligned}
\text{The prime factors of } 36 &= 6 \times 6 \\
&= (3\times2) \times (3\times2) \\
&= (2\times2) \times (3\times3) \\
&= 2^2 \times 3^2
\end{aligned}
$$

VOCABULARY

TERMS

product
factor
composite
prime
Sieve of Eratosthenes
exponential notation

GETTING STARTED

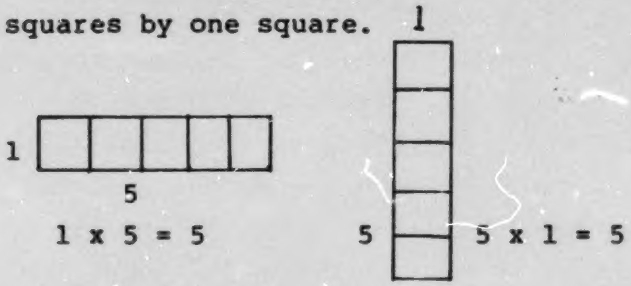
Write 24 on the chalkboard. Under it write 24 = 2 x 12. Say that we call 2 and 12 factors of the product 24. Try this with other numbers, e.g., 14, 8. Ask a student for two factors of 16. Write 16 = ☐ x ☐ on the chalkboard. Repeat using 20, 36 and 48. Say that any number is a result of mutliplying at least two factors.

Make a set of cards with one number from 1 through 100 on the front of the card and the factors of the number on the back, e.g., 12 1,2,3,4,6,12. Ask a student to pick a card and say the factors of the number. Turn the card over to reveal all the factors to see if the student is correct. Ask this student to ask someone else to come and choose a card. Repeat the procedure several times. Stress that students should use the original number and 1 as factors.

7.01.09

7.01.09.01 **determine whether a number is prime or composite**

STRATEGY 1: Prepare a set of twenty squares. Use an overhead projector or chalkboard to display the squares. Ask a student to take six squares and arrange them to form a rectangle, e.g., two squares by three squares. Write 2 x 3 = 6 on the chalkboard. Ask another student to arrange the squares another way, e.g., one square by six squares. Write 1 x 6 = 6. Elicit from the students that 2 x 3 = 6 is the same shape as 3 x 2 = 6, and 1 x 6 = 6 is the same shape as 6 x 1 = 6. Ask the student to list the factors of 6 on the chalkboard. Say, "Have all of the factors in our drawings been shown?" Ask another student to take five squares and arrange them to form a rectangle, e.g., one square by five squares. Write on the chalkboard 1 x 5 = 5. Ask another student to rearrange the five squares, e.g., five squares by one square.



Say that the arrangement of 1 x 5 = 5 is the same as 5 x 1 = 5. Ask the student to list the factors of five. Say, "Have we used all the factors in our drawings?" Repeat this procedure with 7, 13. Ask the students what is the same about the factor shapes of 5, 7 and 13 that is different from the factor shapes of 6. Say to the students that a number with only two different factors, itself and one, is a prime number. Say to the students that we call all other numbers composite numbers.

Prepare a sheet like this for each student.

7.01.09

STRATEGY 2:

Sieve of Eratosthenes

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Say, "Starting with the number 4, cross out all even numbers. Then, cross out all multiples of 3, multiples of 5, multiples of 7, multiples of 9, multiples of 13. Let's look at the numbers that are left. First, let's look at 2. What are the factors of 2? (2 and 1) What are the factors of 3?" (3 and 1) Repeat this procedure with several other numbers. Say, "What do you notice about all of these numbers?" (The only factors are the number itself and 1.) Say that we call these numbers prime numbers. Ask the students to look at the number 1. Say, "One is a special number. We don't call one a prime or a composite number." Draw a box around 1. Say, "What do we call the numbers that we have crossed out?" (composite numbers)

7.01.09.02 **List and order the prime factors of a number**

STRATEGY 1: Write the number 16 on the chalkboard. Say, "In order to find the prime factors of a number, we divide by the smallest prime number which will divide into the number an exact number of times. (2 divides 16 exactly 8 times.) Two is our first prime factor. The quotient 8 is now divided by the smallest prime factor which again is 2. The quotient four can also be divided an exact number

STRATEGY 1: cont'd of times. Since we are left with a quotient of 2, which is a prime number, our problem is completed."

Ask a student to come to the chalkboard and write the following, e.g., 2 x 2 x 2 x 2 are the prime factors of 16 or 16 = 2 x 2 x 2 x 2. Say, "Since the prime factors are all the same, the order in which we write them is inconsequential. Usually, we order prime factors in counting order."

Repeat this procedure using other numbers.

STRATEGY 2: Write the number 30 on the chalkboard. Say that 30 is not a prime number since it can be written as the product of two whole numbers other than itself and 1. Say, "We are going to search for a factor of 30 that is a prime number. Since 30 is an even number, the smallest prime number, 2, is a factor." Write on the chalkboard, 30 = 2 x 15. Say, "Is 15 a prime number? (No) We must now write 15 as a product of a prime number." (15 is divisible by 5.) Write 15 = 5 x 3 on the chalkboard. Say, "Three is also a prime number, so the factored form of 30 is 30 = 2 x 5 x 3." Say that prime factors are listed in counting order. Ask a student to come to the chalkboard, and rewrite the prime factors of 30 in order, e.g., 2 x 3 x 5. Say that the factors and product are the same, only the order is different.

7.01.09.03 **express prime factors in exponential notation**

STRATEGY 1: Ask the students to write the prime factors of 56. Ask one student to come to the chalkboard and write the factors.

e.g.,

56 = 2 x 28 = 2 x 2 x 14 = 2 x 2 x 2 x 7

Say that the prime number two appears three times.

STRATEGY 1: cont'd

Say, "Can anyone think of another way to write 2 x 2 x 2? (exponents or 2^3) Does the result of 2^3 stay the same as 2 x 2 x 2? How are the two forms different?" (The form of 2^3 is shorter.) Ask a student to come to the chalkboard and write the number 56 as the product of prime numbers using exponential form, e.g., $56 = 2^3 \times 7$.

Repeat this procedure using 12 and 60.

STRATEGY 2:

Display on the chalkboard or an overhead projector the following.

e.g.,

$$16 = 2 \times 2 \times 2 \times 2$$

Say, "The example shows the prime factors of 16. The way it is written is long and repetitious. We have a shorthand for multiplication of factors that appear more than once. Does anyone know what we call this shorthand notation?" (exponent or power) Ask a student to come to the chalkboard and show where the exponent is placed.

e.g.,

2^4 exponent

Say, "We call this exponential notation."

Write 45 on the chalkboard. Ask a student to come to the chalkboard and write it as a product of prime numbers.

e.g.,

$$\begin{aligned} 45 &= 3 \times 15 \\ &= 3 \times 5 \times 3 \\ &= 3 \times 3 \times 5 \end{aligned}$$

Ask another student to write it in exponential notation.

e.g.,

$$\begin{aligned} 45 &= 3 \times 3 \times 5 \\ &= 3^2 \times 5 \end{aligned}$$

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Repeat this procedure using 36 and 144.

P.O. 7.01.09.01

NAME:______________________________ CLASS:________ DATE:________

Priming My Way Home

Can you find your way home? Move from start to home by following a path of prime numbers. Begin at the starting point with "2". Find the next prime number by moving vertically, horizontally or diagonally. Join each newly found prime number in order to form a chain of prime numbers from "start" to "home."

						81	44	36	26	18	76	80	40	28	33	52	22	86
						27	51	68	22	89	41	31	89	13	17	68	50	75
						70	97	23	61	83	46	27	34	96	43	77	54	36
Start →	②	29	67	57	10	52	7	24	62	10	65	20	92	[illegible]	61	69	93	10
	22	90	23	83	97	17	19	9	84	48	52	14	58	17	73	82	48	24
	65	58	86	42	24	81	45	51	34	22	35	69	50	27	83	97	2	45
	74	42	68	34	55	16	82	87	28	84	14	39	85	46	26	10	5	12
	52	25	33	20	6	39	46	32	14	90	58	40	12	28	42	23	29	88
													72	48	71	43	18	14
													58	27	97	32	51	57
													70	44	43	86	84	27
													84	18	31	23	17	64
													39	36	60	6	29	㊼

↓

Home

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P.O. 7.01.09.02

NAME:______________________________ CLASS:________ DATE:_______

Write the prime factors of each number.
Order the factors.

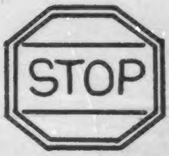
	Factors	Order of Factors
1) 90 =	2 x 3 x 5 x 3	2 x 3 x 3 x 5
2) 36 =	________	____________
3) 100 =	________	____________
4) 144 =	________	____________
5) 210 =	________	____________

P.O. 7.01.09.03

NAME: ____________________ **CLASS:** ________ **DATE:** ________

Write the prime factor of each number using exponential notation.

	Number	=	Factors	=	Factors in exponential form
1)	12 =		2 x 2 x 3		$2^2 \times 3$
2)	60 =		________		____________
3)	24 =		________		____________
4)	311 =		________		____________
5)	78 =		________		____________



7.01.10 SQUARES AND SQUARE ROOTS

(2-3 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

7.01.10.01A find the square of a number;

e.g.,

5 squared is 25

(5 x 5 = 25)

$5^2 = 25$

7.01.10.02A list the squares of numbers;

e.g.,

Numbers	Squares
1	1
2	4
3	9
.	.
.	.
.	.
15	225
.	.
.	.
.	.

7.01.10.03A identify the square root of a number that is a perfect square;

e.g.,

The square root of 16 is 4.

4 is the square root of 16.

$\sqrt{16} = 4$

7.01.10.04B find the square root of a decimal fraction which is a perfect square;

e.g.,

$$\sqrt{.64} = .8$$

7.01.10.05A solve verbal problems related to skills learned in this module.

e.g.,

A garden plot is 49 m^2. How long will one side be?

area = 49 m^2

side = 7 m

7.01.10

VOCABULARY

TERMS	SYMBOLS
radical	
exponent	
square	
decimal fraction	
common fraction	
perfect square	n^2
square root	$\sqrt{}$

GETTING STARTED

Play "Tic-Tac-Toe" with composite numbers. Draw a tic-tac-toe diagram on the chalkboard.
e.g.,

A	B	C
D	E	F
G	H	I

Divide the class into two teams, X's and O's. Say, "This game requires that you name prime numbers." Choose a student on one team to say a prime number. If this number is prime , e.g., 5 or 7, the team will score an X or an O in the box they call. If the student calls a composite number, e.g., 24 or 62, they lose their turn. Repeat this with each team. When a team fills any row in any direction, that team is the winner.

Write this chart on the chalkboard.
e.g.,

Common Fraction	Decimal Fraction
1) $\frac{6}{10} \times \frac{3}{10} = \frac{18}{100}$	$.6 \times .3 = .18$
2) $\frac{8}{10} \times \frac{4}{10} = \frac{32}{100}$	$.8 \times .4 = .32$
3) $\frac{9}{10} \times \frac{3}{10} = \square$	$.9 \times .3 = \square$
4) $\frac{7}{10} \times \frac{5}{10} = \square$	$\square \times \square = \square$

Ask several students to read examples 1 and 2, then to fill in examples 3 and 4. Elicit from the class that the product of tenths times tenths is hundredths. Repeat this procedure with other examples.

7.01.10

7.01.10.01 **find the square of a number**

STRATEGY 1: Use large size graph paper on the chalkboard or an overhead projector. Draw the following figures.

Figure 1

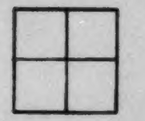


Figure 2

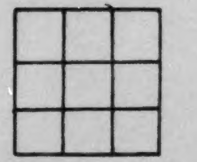
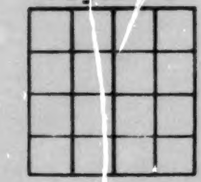


Figure 3



Ask a student to count the squares in the first figure, and another student to translate Figure 1 into words and symbols.

e.g.,

2 squares by 2 squares = 4 squares
2 squared is 4
$2 \times 2 = 4$
$2^2 = 4$

Repeat this procedure for Figure 2 and Figure 3.

STRATEGY 2: Write this chart on the chalkboard.

e.g.,

Number	Number x Number	Number Squared	Product
5			

Ask a student to complete each column.

e.g.,

5	5×5	5^2	25

Repeat this procedure for 10, 3, 11, and 9. Then fill in different parts of the chart and ask the students to complete the empty columns.

7.01.10.02 **list the squares of numbers**

STRATEGY 1: Ask the students to square the first 15 counting numbers by listing them on a chart like the one

STRATEGY 1:
cont'd

displayed on the chalkboard. Ask several students to complete the chart on the chalkboard.

Number	Number Squared
1	1
2	4
3	9
.	.
.	.
.	.
.	.
10	100
.	.
.	.
.	.
15	225

Ask the students to square the first 10 multiples of 10 completing a chart similar to the one diplayed on the chalkboard. Ask several students to complete the chart at the chalkboard.

Number	Number Squared
10	100
20	400
30	900
.	.
.	.
.	.
.	.
.	.
100	10,000

Ask the students to compare the squares of the first 10 counting numbers and the squares of the multiples of 10. Refer to the completed chart on the chalkboard.

Squares of Numbers 1-10	Squares of Multiples of 10-100
1	100
4	400
9	900
16	1,600
25	2,500
36	3,600
49	4,900
64	6,400
81	8,100
100	10,000

Directing the students to compare the tables, elicit that the squares of the multiples of 10

STRATEGY 1: cont'd — through 100 are 100 times the squares of the first 10 counting numbers.

STRATEGY 2: Construct flashcards of the first 15 counting numbers.

e.g., $\boxed{1^2}$ $\boxed{2^2}$ $\boxed{3^2}$ $\boxed{4^2}$ $\boxed{5^2}$

Say, "How are 3^2 and 3 x 2 different? The exponent two says that the base 3 is a factor two times, e.g., $3^2 = 3 \times 3$." Repeat this with the other number cards.

7.01.10.03 **identify the square root of a number that is a perfect square**

STRATEGY 1: Introduce the symbol $\sqrt{}$ meaning square root. Write on the chalkboard $\sqrt{36} = 6$ means the square root of 36 is 6 because 6 x 6 = 36. Repeat this with other perfect squares.

e.g., $121 = \boxed{}$ $49 = \boxed{}$ $81 = \boxed{}$

STRATEGY 2: Write the following chart on the chalkboard. Say that n means the number, n^2 means the perfect square, $\sqrt{n}$ means square root.

The Square Root of a Perfect Square

n	n^2
	9
	16
	25
	49
	64
	81
	100
	121
	144
	169
	196
	225
	256

STRATEGY 2: cont'd

Ask the students to copy the chart into their notebooks. Ask the students to fill in the number column. Ask several students to complete the chart at the chalkboard.

7.01.10.04 find the square root of a decimal fraction which is a perfect square

STRATEGY 1:

Write on the blackboard.

e.g., $\sqrt{\frac{25}{100}} = \square$

Ask a student to find the square root of the numerator, then find the square root of the denominator.

e.g., $\frac{\sqrt{25}}{\sqrt{100}} = \frac{5}{10}$

Ask the students to prove this by multiplication.

e.g., $\frac{5}{10} \times \frac{5}{10} = \frac{25}{100}$

Translate this sentence into decimal notation.

e.g.,

$.5 \times .5 = .25$

Repeat this procedure with the following examples.

e.g.,

(1) $\frac{\sqrt{36}}{\sqrt{100}} = \frac{6}{10}$ $\frac{6}{10} \times \frac{6}{10} = \frac{36}{100}$

$\sqrt{.36} = .6$

(2) $\frac{\sqrt{49}}{\sqrt{100}} = \frac{7}{10}$ $\frac{7}{10} \times \frac{7}{10} = \frac{49}{100}$

$\sqrt{.49} = .7$

7.01.10

STRATEGY 1: cont'd

Then continue to find the square root using only the decimal form.

e.g., $\sqrt{.64}$, $\sqrt{.16}$, ...

STRATEGY 2:

Prepare a set of cards with a decimal fraction that is a perfect square on each.

e.g., $\boxed{\sqrt{.49}}$ $\boxed{\sqrt{.64}}$ $\boxed{\sqrt{.16}}$

Say, "To find the square root of a number, we need two identical factors, which when multiplied, will give us this decimal." Ask a student to select a card and find the square root. Say, "These decimals are all hundredths. What is the place value of the two decimal factors which will give us hundredths?" (tenths) Repeat this procedure using all of the cards.

7.01.10.05 **solve verbal problems related to skills learned in this module**

STRATEGY 1:

Write this problem on the chalkboard.

> Jose and Maria are having an argument about whose garden is larger. They both insist they're right. Jose's garden is 6 ft. by 6 ft. Maria's garden is 9 ft. by 4 ft. How can you determine who is correct?

Ask half of the students to find the area of Jose's garden. Ask the other half of the students to find the area of Maria's garden.

e.g.,

Jose's Garden	Maria's Garden
$A = S^2$	$A = lw$
$= 6 \times 6$	$= 9 \times 4$
$= 36$ sq. ft.	$= 36$ sq. ft.

STRATEGY 1: cont'd

Say, "Who has the bigger garden?" (Neither, as both gardens are the same size.) Continue with other problems involving squares and square roots.

STRATEGY 2:

Ask the students to read this problem.

> Mike has a square board which measures .5 of a foot on each side. How large should a piece of foil be so it can cover the top of the board?

Ask a student to state the formula Mike would use to find the answer. ($A = S^2$) Remind the students to use problem solving techniques to determine their answers. Continue this procedure using other problems which involve squares and/or square roots.

P.O. 7.01.10.01

NAME: ____________________ CLASS: ________ DATE: ______

Find the square of the following numbers.

6 squared is $6^2 =$ $6 \times 6 =$	3 squared is $3^2 =$ $3 \times 3 =$	11 squared is $11^2 =$ $11 \times 11 =$	14 squared is $14^2 =$ $14 \times 14 =$
$12^2 =$	$9^2 =$	$10^2 =$	$8^2 =$
$15^2 =$	$24^2 =$	$6^2 =$	$17^2 =$
$1.1^2 =$	$1.5^2 =$	$20^2 =$	$18^2 =$

P.O. 7.01.10.02

NAME: ______________________ CLASS: __________ DATE: ________

State the square for each number.

1 1

5 ______

8 ______

15 ______

6 ______

12 ______

7 ______

10 ______

3 ______

2 ______

13 ______

14 ______

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P.O. 7.01.10.03

NAME: ______________________________ CLASS: __________ DATE: ________

Find the square root of each perfect square.
Color or shade it in.

PLAY WHIZO!

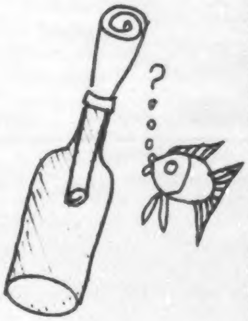
W	H	I	Z	O
18	49	24	16	12
32	14	28	46	50
6	84	★	45	10
25	80	42	90	64
200	81	100	225	56

P.O. 7.01.10.04

NAME:______________________________CLASS:__________DATE:________

Find the square root of each. Determine its letter value in the code box. Write the message.

Can You Find the Message?



$\sqrt{.16}$ $\sqrt{.64}$ $\sqrt{.81}$ $\sqrt{.25}$ $\sqrt{.49}$ $\sqrt{.36}$ $\sqrt{.16}$

answer: .4 ____ ____ ____ ____ ____ ____

letter: S ____ ____ ____ ____ ____ ____

$\sqrt{.25}$ $\sqrt{.49}$ $\sqrt{.36}$

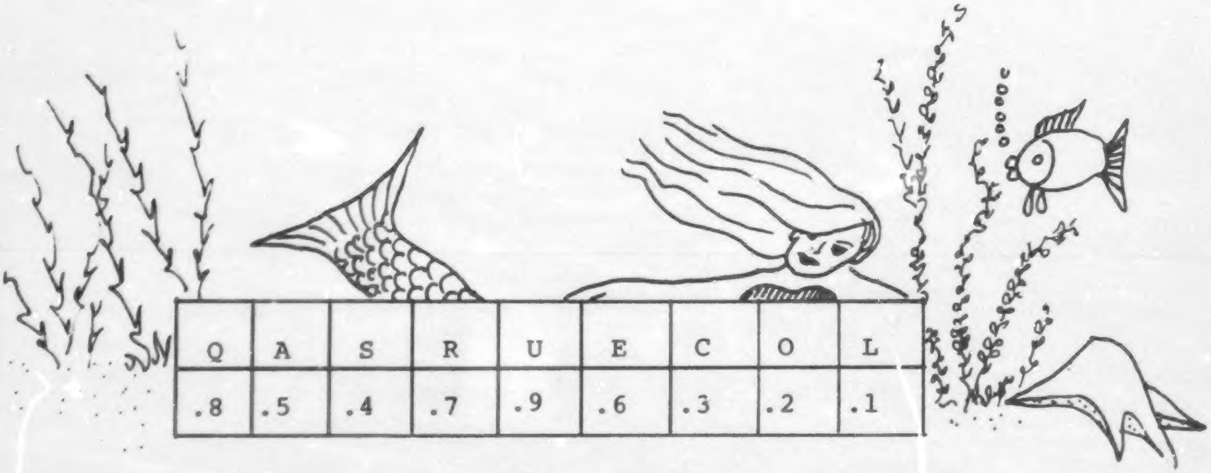
____ ____ ____

____ ____ ____

$\sqrt{.09}$ $\sqrt{.04}$ $\sqrt{.04}$ $\sqrt{.01}$

____ ____ ____ ____

____ ____ ____ ____



Q	A	S	R	U	E	C	O	L
.8	.5	.4	.7	.9	.6	.3	.2	.1

P.O. 7.01.10.05

NAME: ______________________ CLASS: ________ DATE: ________

Solve each problem. Show all work.

1) The side of an area of land is 12 m. How much fencing will be needed to enclose the land if all the sides measure the same length?

2) Find the length of the side of a square national forest whose area is 6400 km^2?

3) A microscopic slide is divided into squares, the area of each square is 25 m^2. What is the length of one side of the square?

4) The students participating in the school fair must build their booths in the schoolyard which is a square. If one side is 90 ft. in length, how much space is available for the fair?